

Enhancing Decision Quality in Multi-Criteria Decision Making through CISDAC-WSM Algorithm

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Abstract

This paper introduces CISDAC-WSM, an innovative extension of the Weighted Sum Method (WSM) within the Multi-Criteria Decision Making (MCDM) framework. CISDAC-WSM integrates principles from Principal Component Analysis (PCA) to enhance decision outcomes by identifying the Most Significant Discriminating Axes. The algorithm operates under the assumption that alternative rankings should exhibit a monotonic trend in the scores of the Most Significant Discriminating Axes. In addition to leveraging PCA, CISDAC-WSM introduces an interval-based conflict resolution mechanism for alternatives with similar rankings. Unlike traditional outranking algorithms like PROMETHEE, CISDAC-WSM focuses on comparing each alternative only with those ranked superior, resulting in more targeted evaluations. Empirical comparisons and performance evaluations demonstrate that CISDAC-WSM consistently outperforms existing MCDM algorithms. Through its emphasis on identifying the Most Significant Discriminating Axes and the innovative conflict resolution strategy, the algorithm showcases enhanced decision-making capabilities and efficiency. While CISDAC-WSM is an extension rather than an entirely new algorithm, its contributions lie in refining established methods, incorporating PCA insights, and offering a more localized approach to outranking. This makes it a promising advancement in the field of MCDM, presenting a refined and innovative technique for achieving more informed and effective decision outcomes across various domains.

Keywords: Multi-Criteria Decision Making (MCDM), Decision Science, Weighted Sum Method (WSM), Principal Component Analysis (PCA), Optimization, Utility Theory, Operations Research, Monotonic Trend, Performance Metrics, Comparative Analysis, Most Significant Discriminating Axes.

1. Introduction

In complex decision-making instances with several alternatives, the decision making process becomes more complex and sophisticated [9]. MAUT, an extension of traditional utility theory, enables decision-makers to assign utility values to different traits and calculate total utility measurements [14], making this field critical in providing a reliable approach for making optimal decisions [10].

Multi-Criteria Decision Making (MCDM) broadens this paradigm by introducing systematic statistical

techniques to weighing and prioritizing attributes based on their relative importance [12].

This realization enables decision-makers to make educated decisions that are consistent with their goals within the larger framework of MAUT. Multi-Attribute Utility Theory (MAUT) is based on the idea that evaluating alternatives is a systematic process with discrete steps [25], [18]:

- Define alternatives and attributes, determine evaluations of each alternative on individual attribute.
- Verify preferential and utility independence conditions.
- Assign relative weights to attributes and derive the multi-attribute utility function.
- Aggregate weights and multi-attribute evaluations.
- Perform sensitivity analyses and make recommendations.

While, Bell et al. (1977) and Keeney and Raiffa (1993) developed a basic additive utility function to assess the relative significance of alternatives for decision-makers [3, 11], addressing conflicting criteria in Multi-Attribute Utility Theory (MAUT) and Multi-Criteria Decision Making (MCDM) remained a challenge, as conflicting criteria develop when the best decision for one contradicts the best value for a different criterion [3].

Strategies for dealing with contradictory criteria take a sophisticated approach, focusing on maximum utility while accepting minimal trade-offs [1], making sensitivity analysis and scenario evaluations critical tools for determining the impact of different criteria weights on the final decision [11]. Managing contradicting criteria in MAUT requires a thorough knowledge and a strategic approach to balancing competing objectives.

2. Literature Review

In the realm of Multi-Attribute Utility Theory (MAUT) and Multi-Criteria Decision Making (MCDM), the application areas of these methods are extensive [12], encompassing various domains such as supplier selection, technical evaluation of tenders [15], selection of cooking devices [18], assessment of service quality [13], and the evaluation of renewable energy projects [1,14].

However, in specific decision problems, the selection of the most appropriate MCDM method becomes a challenge, lacking clear guidelines [21-23].

Hence, this issue has been a subject of study for decades. There are many MCDM methods in the literature, as PROMETHEE [4, 5, and 24], AHP [20], ELECTRE [19], etc. In this work, we focus on multi-MOORA [2, 6, 7], TOPSIS [8] and VIKOR [16, 17]. Multi-MOORA applies aggregation operators, while VIKOR operates calculating distances to “ideal” or “reference” points and PROMETHEE applies an outranking method. We selected these methods for comparison because they have the same input and all of them rely on a normalization procedure.

3. Methodology

The methodology employed includes a review of existing Multi-Criteria Decision Making (MCDM) algorithms, with a particular focus on VIKOR, Multi-MOORA and established outranking algorithm such as PROMETHEE.

Subsequently, an algorithm extension named CISDAC-WSM was conceptualized, integrating insights from PCA and introducing an interval-based conflict resolution mechanism.

The benchmarking process assessed the performance of CISDAC-WSM against existing MCDM algorithms, such as PROMETHEE, VIKOR, and Multi-MOORA, focusing on key metrics like computation time, memory requirements, and correlation coefficients. Sensitivity analysis assessed the algorithm's consensus with other algorithms by evaluating its correlation with them, using Kendall tau and Spearman's rank correlation coefficients, measuring the ranking "agreement" between methods.

Feature weight distribution analysis, studied the effects of weight distribution on the correlation coefficient.

Lastly, a parametric study explored the effects of changing parameters on correlation coefficients, determining optimal ranges for correction measure and conflict radius.

Table 1 shows popular MCDM algorithms and used for comparative analysis.

Table 1 Popular MCDM Algorithms Used for Comparative Analysis

PROMETHEE II	The PROMETHEE process involves the evaluation of the alternatives by making pairwise comparisons to determine the dominating or better alternative in each case using preference functions as given by Brans and Vincke.[4,5,24]
VIKOR	VIKOR stands for VlsekriterijunskoKOMPromisnoRangiranje , basically deals with Multi-criterion optimization and finding optimized solutions along with there is a compromise solution agreement established by mutual agreement.[16,17]
Multi-MOORA	Multi-MOORA constructs a ranking departing from three calculations: the “Ratio System”, the “Reference Point” and the “Full Multiplicative Form of Multiple Objectives” [6, 7].

4. CISDAC-WSM (Introduced Algorithm)

CISDAC-WSM (Conflict Interval based Significant Discriminating Axis Corrective -Weighted Sum Method), an extension of the Weighted Sum Method (WSM) in Multi-Criteria Decision Making (MCDM) proposed by us, incorporates Principal

Component Analysis (PCA) insights for improved decision outcomes. The algorithm introduces an interval-based conflict resolution mechanism, outranking alternatives based on conflict degree and axes scores.

5.1 Algorithm

Step 1: The feature values are normalized for all alternatives.

$$\text{Normalised feature } x_i(a) = \begin{cases} \frac{f_i(a) - \mu_i}{\sigma_i}, & i \in \text{Maxim}(D), \text{ or benefit criteria} \\ \frac{\mu_i - f_i(a)}{\sigma_i}, & i \in \text{Minim}(D), \text{ or cost criteria} \end{cases} \quad (1)$$

Step 2: Find Weighted Sum score and Conflict Intervals.

$$S(a) = \sum_{j \in F} w_j x_j(a) \quad (2)$$

$$I(a) = (S(a) - v * \beta, S(a) + v * \beta), \text{ where } \beta = \sqrt{\sum_{j \in F} w_j x_j^2(a) - S(a)} \quad (3)$$

Where v- Conflict Radius, F is the feature set of the alternative

Step 3: Rank According to WSM score.

Total Ordered Set with utility measure S, $B = \{b_1 > b_2 \dots > b_m\} = (A, <)_S$,

Where $b_i > b_k \Leftrightarrow S(b_i) > S(b_k)$, B is the set of alternatives A ,ordered in the decreasing order of the utility measure S

Step 4: Find the Maximum one sided Conflict for each alternative.

$$c_i = \max_{j < i} (d(I(b_i) \cap I(b_j))) \quad (4)$$

Step 5: Principal Components are found and taken in the decreasing order of Eigenvalues.

$$\Lambda = \left\{ (\lambda, \bar{v}) \mid \bar{v} \text{ is the Principal Component of Normalised Decision Matrix with eigenvalue } \lambda \right\} \quad (5)$$

Normalized Eigenvalue $\lambda' = \frac{\lambda}{\sum_{(\mu, v_\mu) \in \Lambda} \mu}$ (6) Derived weight of principal component $w_\lambda = v_\lambda \cdot w_F$ (7)

Normalized Weight of principal component $w'_\lambda = \frac{w_\lambda}{\| [w_\lambda]_\Lambda \|}$ (8) Magnitude of principal component weight vector $\| [w_\lambda]_\Lambda \| = \sqrt{\sum_{(\mu, v_\mu) \in \Lambda} w_\mu^2}$ (9)

Most Significant Discriminating Axes Subset, $(\Omega, >)_{(\lambda, w_\lambda)}$ with comparison measure taken to be the significance of the principal component, (10)

$$\langle \lambda; w_\lambda \rangle = |\lambda' * w'_\lambda| \sum_{(\lambda, v_\lambda) \in \Omega} \langle \lambda; w_\lambda \rangle > \Phi,$$

where $\Omega \subseteq \Lambda$ and significance threshold, $\Phi \in [0.5, 1]$, Taking eigenvalue, eigenvector pairs in the decreasing order of eigenvalues. Change in threshold doesn't affect ranking continuously due to the discretized, discontinuous and skewed significance values of the discriminating axes.

Step 6: For each alternative the score correction is then applied $S'(b_i) = S(b_i) + r * \log(v * c_i) * ([v_\lambda \cdot F(b_i)]_\Omega \cdot [w_\lambda]_\Omega)$, where $b_i \in A$ (11)

$$[v_\lambda \cdot F(b_i)]_\Omega \cdot [w_\lambda]_\Omega = \sum_{(\lambda, v_\lambda) \in \Omega} w_\lambda (v_\lambda \cdot F(b_i)) \quad (12)$$

Where $F(b_i)$ is feature vector of b_i and v_λ is the principal component corresponding to eigenvalue λ . Hence $v_\lambda \cdot F(b_i)$ is the projection of the feature vector along v_λ , $[v_\lambda \cdot F(b_i)]_{(\lambda, v_\lambda) \in \Omega}$ is the resultant feature vector along the most significant axes. $[w_\lambda]_\Lambda$ is the weight vector of the most significant discriminating axes.

\therefore decision coefficient, $d(a) = S'(b_i)$

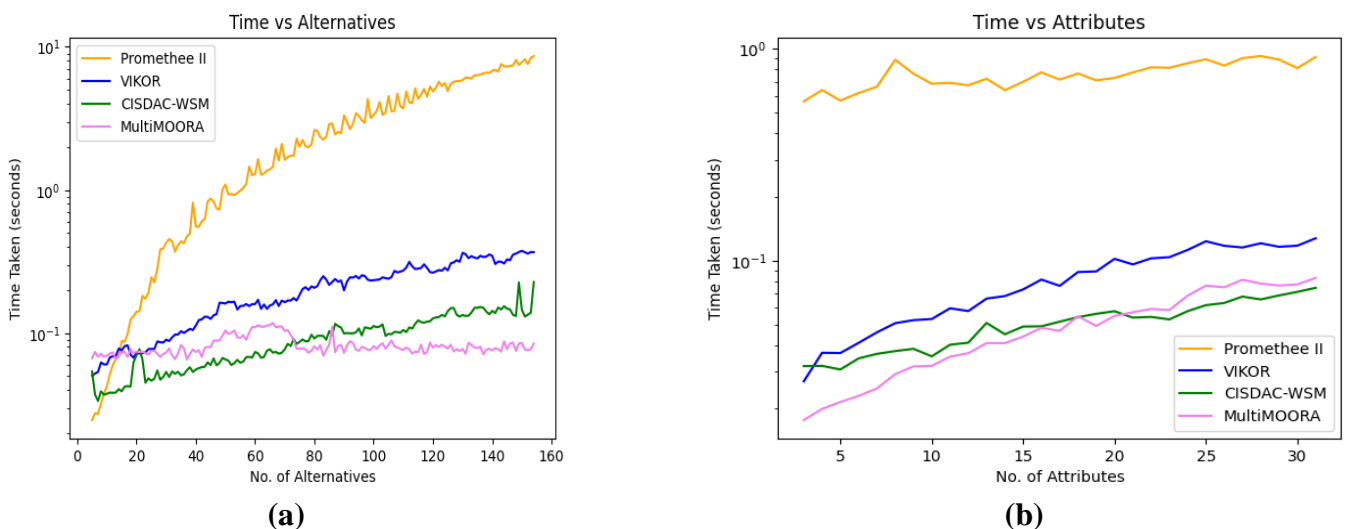
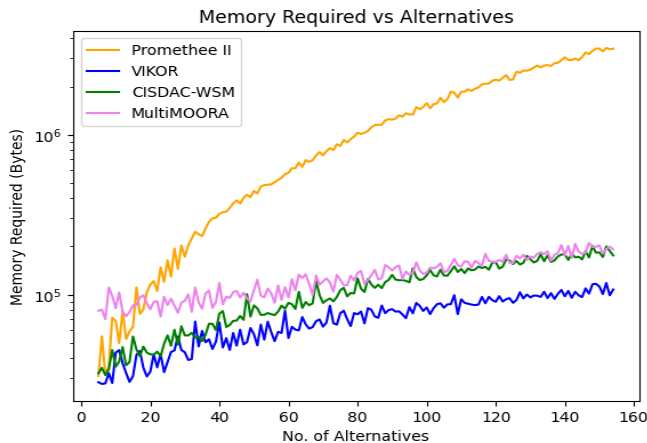


Figure 1 Computational Time (Seconds) Required for Different MCDM Algorithms

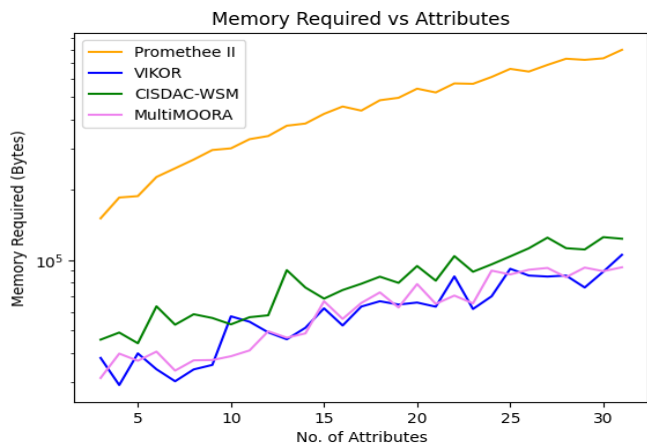
5. Comparative Analysis

5.1 Computational Time

In this part, the computational time frames for each method across situations with various numbers of choices and characteristics are examined and analyzed (see Figure 1). Our approach beat VIKOR and PROMETHEE II in cases with several alternatives, displaying higher computational efficiency. Our algorithm performed similarly to Multi-MOORA, suggesting its appropriateness for a wide range of decision-making procedures. This competitive performance, particularly against known approaches such as VIKOR and PROMETHEE II, demonstrates our algorithm's efficiency and resilience in dealing with complicated decision-making situations involving an increasing range of characteristics.



(a)



(b)

Figure 2 Computational Memory Requirement (Bytes) for Different MCDM Algorithms

5.2 Memory Usage

A notable finding emerges from a thorough investigation of relative memory requirements, demonstrating that the slope of our algorithm's memory consumption is far flatter than that of other algorithms when confronted with a variable number of characteristics. Using regression analysis to extrapolate this tendency, we propose a hypothesis that memory needs tend to become more affordable as the number of characteristics increases (Figure 2). Furthermore, given the amount of alternatives, it is clear that VIKOR and Multi-MOORA serve as upper and lower limits, respectively, for CISDAC-WSM. This result emphasizes CISDAC-WSM's intermediate position, implying nuanced memory efficiency across scenarios with varying numbers of choices.

Kendall Tau Correlation (100 alternatives)				
	PROMETHEE II	VIKOR	Multi-MOORA	CISDAC
PROMETHEE II	1.000000	0.975354	0.945859	0.977374
VIKOR		1.000000	0.956768	0.985859
Multi-MOORA			1.000000	0.947475
CISDAC				1.000000

(a)

Spearman Rank Correlation (100 alternatives)				
	PROMETHEE II	VIKOR	Multi-MOORA	CISDAC
PROMETHEE II	1.000000	0.998368	0.994167	0.998608
VIKOR		1.000000	0.995824	0.999220
Multi-MOORA			1.000000	0.994287
CISDAC				1.000000

(b)

Figure 3 Correlation Coefficient Test for Sensitivity Analysis for MCDM Algorithms

5.3 Sensitivity Analysis

Through sensitivity analysis, it becomes clear that CISDAC-WSM has a significant correlation with all algorithms, as evidenced by both Kendall Tau and Spearman's Rank correlation coefficients.

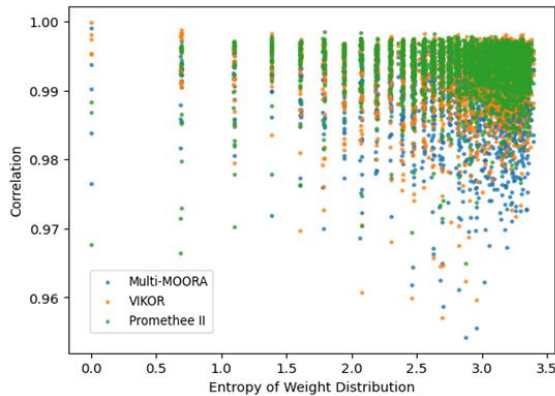
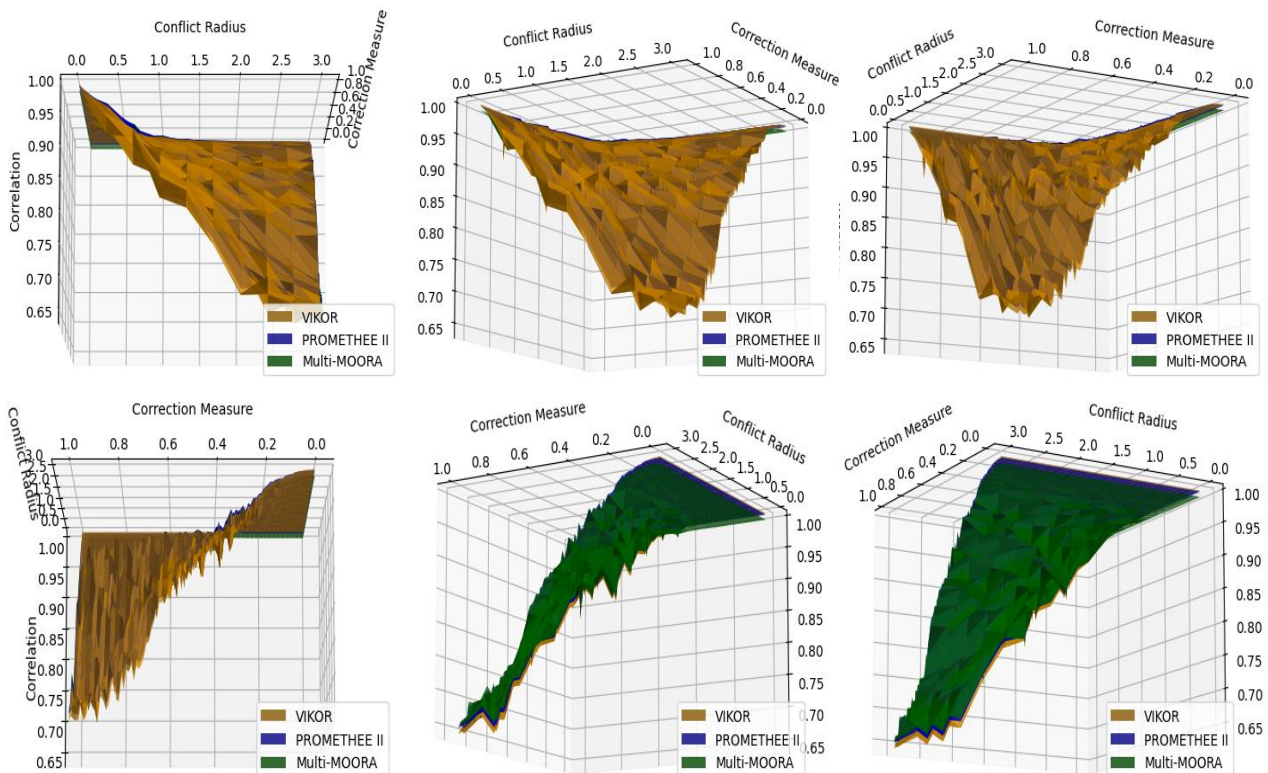


Figure 4 Variation of Correlation with CISDAC-WSM with Change in Entropy of Feature Weight

A correlation greater than 0.9 indicates that the output of our algorithm closely follows the

established trend of preference. We also observe, CISDAC-WSM has a stronger alignment with VIKOR, followed by PROMETHEE II. This correlation insight highlights the consistency and compatibility of CISDAC-WSM with known algorithms, notably in capturing and expressing general preferences in decision-making environments, as seen in Figure 3.

In our analysis of the effect of feature weight distribution on the correlation coefficient, we observe that ideal results are obtained when there is a more balanced and uniform weight distribution. Deviations from this trend occur when the distribution becomes more skewed, underscoring the significance of a balanced feature weight distribution in achieving optimal correlation, providing valuable insights for decision-makers seeking to enhance the algorithm's performance by carefully considering the distribution of weights assigned to different features are shown in Figure 4.



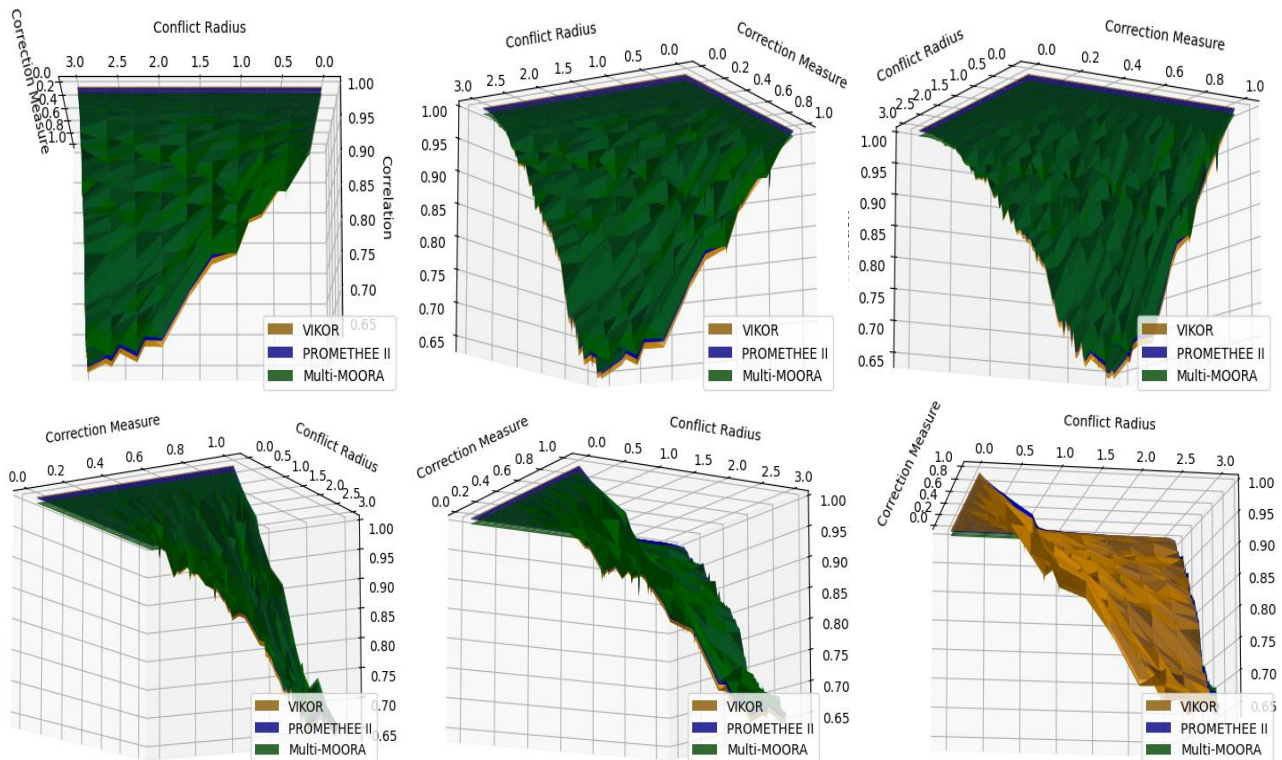


Figure 5 Parametric Surface for Varying Conflict Radius and Correction Measure in CISDAC-WSM Algorithm

6. Parametric Analysis

In the parametric study, we systematically examined the effects on correlation with changing parameters. Our findings indicate that ideal results were achieved when the correction measure ranged up to 0.6 and the conflict radius extended up to 1.5. This understanding of the appropriate ranges for these parameters aids in the fine-tuning and successful application of CISDAC-WSM in decision-making scenarios. Figure 5 depicts the parametric surface for changing conflict radius and correction measure in the CISDAC-WSM algorithm.

The study offers useful advice on parameter choices that are consistent with the algorithm's performance, assuring its flexibility and efficacy under different scenarios.

7. Results & Discussion

The comparative analysis demonstrated that CISDAC-WSM outperformed previous MCDM algorithms. In terms of computing time, the approach was efficient, especially in cases with a

growing number of features. The memory requirement study revealed a more economical tendency for CISDAC-WSM, particularly as the number of characteristics rose, establishing it as an intermediate memory efficiency between VIKOR and Multi-MOORA. Sensitivity study revealed strong connection with other algorithms, with Kendall Tau and Spearman's Rank connection values more than 0.9, showing persistent preference alignment. Furthermore, the parametric analysis found optimal outcomes with a corrective measure of 0.6 and a conflict radius of up to 1.5.

The analysis of feature weight distributions reveals that CISDAC-WSM produces the best results for balanced and uniform weight distributions, with discrepancies seen for skewed distributions.

Conclusion

In conclusion, the presented CISDAC-WSM algorithm is an improvement to existing MCDM approaches, demonstrating increased efficiency, lower memory requirements, and consistent

preference alignment with other algorithms. Its performance in computing time, sensitivity analysis, and parametric study make it a viable tool for decision-makers seeking optimal outcomes in a variety of settings. The algorithm's versatility and resilience make it a significant contribution to the field of Multi-Criteria Decision Making.

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