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Infinite Potential Box Problem in Fibonacci Calculus

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Abstract
We consider the q_1, q_2-deformed formalism constructed with some elements of Fibonacci calculus to study the 1D infinite potential box problem where 1-D Schrödinger equation is reframed with well-known modified Fibonacci difference operator and has been solved using q-series solution method. In order to do it, at first, all the selective but essential basics are displayed one by one and q_1, q_2-deformed trigonometric functions have been plotted to review and understand the deformed quantum mechanical framework. Hence, on the basis of two parameter deformed algebra, wave functions associated with the particle confined inside the infinite potential box has been obtained. The normalized wave functions and energy expressions have also been obtained for $q_1 = \xi,\ q_2 = -1/\xi$. It is found that all the expressions can be reduced to their conventional form within the limit $q_1 \to 1$, and $q_2 \to 1$.

Keywords: *q_1, q_2-deformed formalism, Fibonacci calculus, 1D infinite potential box, modified Fibonacci difference operator.*

1. Introduction
Quantum calculus, called as q-calculus is an unconventional type of calculus which has no need to use the concept of limit [1-3]. Thus, q-calculus can explore the microscopical physical phenomenon in a more straightforward mathematical framework that describe a physical system in simple and different way. It is also a matter of concern that till the last two- or three-decades mathematicians and physicist tried to develop several q-analogue theories in nearly all the field of physical science from statistical mechanics to astrophysics. Very recently, a number of problems in classical physics has also been studied in Jackson's q-framework [4-10]. Even, as an extension, q-fractional non-linear biological population model can study an innovative q-analysis which is important to understand superconductivity and fiber optics [11]. As Jackson's q-trigonometric functions are not periodic in general, it is difficult to model any physical system up to that mark what classical calculus can do. In order to overcome this situation several deformed frameworks have been introduced to model a number of quantum systems [12-15]. Recently, it is realized that q-calculus can study the physics of microscopic world which is complex and include many-body interaction to understand the nonlinear behavior with modified theories. Now a days, several novel theories has been introduced to describe some deformed versions of well-known quantum physical system by setting modified quantum mechanical postulates. All these approaches describe quantum mechanical oscillator problem with different particle algebra [9-17]. Motivated from the work as described in ref. [12,15-19], we have studied infinite potential box problem

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using two-parameter deformed calculus. It is also important to note that this problem being the very basic problem in quantum chemistry and mathematical physics can be explored further to study different types of potential problems linked to material science.

1.1 Basics of q-calculus

It is essential to introduce a few basics definitions of q-calculus to formulate the two-parameter deformed framework as described in [1-3,17]. However, the basic q-number is defined as,

$$[\alpha]_q = \frac{q^n - 1}{q - 1}, \qquad \dots (1)$$

which is in accordance with the Jackson's q-derivative

$$D_{q,x} = \frac{F(qx) - F(x)}{(q-1)x} . \qquad \dots (2)$$

In that scenario, the one parameter q -deformed Bosonic algebra follows relations

$$AA^\dagger - qA^\dagger A = 1 , \qquad AA^\dagger = [\hat{N}]_q. \qquad \dots (3)$$

Where A's are quantum mechanical step operators. Here, it is important to note that as q tends to 1 , both of these two reduces to their ordinary form. After that, symmetric calculus is introduced to define the q-oscillator algebra in a new mathematical footing where the q-symmetric number was defined by,

$$[\alpha]_{q\leftrightarrow 1/q} = \frac{q^n - q^{-n}}{q - q^{-1}} \qquad \dots (4)$$

with the derivative,

$$D_{q\leftrightarrow 1/q,,x} = \frac{F(qx) - F(q^{-1}x)}{(q - q^{-1})} , \qquad \dots (5)$$

And

$$AA^\dagger - qA^\dagger A = q^{-\hat{N}}. \qquad \dots (6)$$

1.2 Elementary Fibonacci Oscillator Algebras

It is remarkable that different versions of q-deformed quantum mechanics has been introduced to develop a complete deformed theory as like as conventional quantum theory. But theoretical studies using two-parameter q-deformation remains a matter of active research till today. In this section, we shall review the bosonic as well as fermionic oscillator algebra as described in ref. [16, 17, 19] to establish the two-parameter deformed framework to study the present problem. It is important to explain that, Fibonacci oscillators A_m , and the corresponding quantum mechanical creation operator $A_m^\dagger$ follow the relations given by,

$$[A_m, A_n^\dagger] = 0, m \neq n,$$
$$\text{and } [A_m, A_n] = 0,$$
$$\text{where}, m, n = 1,2,3, \dots \qquad \dots (7)$$

$$A_m A_m^\dagger - q_1^2 A_m^\dagger A_m = q_2^{2\hat{N}_m},$$
$$A_m A_m^\dagger - q_2^2 A_m^\dagger A_m = q_1^{2\hat{N}_m},$$
$$\text{and } A_m^\dagger A_m = [\hat{N}_m] , A_m A_m^\dagger = [\hat{N}_m + 1] \qquad \dots (8)$$

Thus, the generalized Fibonacci integers as obtained from the spectrum of deformed bason number operators $[\hat{N}_m]$ or in terms of $A_m, A_n^\dagger$ is found as

$$[\alpha]_{q_1,q_2} = A_m A_m^\dagger = \frac{q_1^{2\alpha} - q_2^{2\alpha}}{q_1^2 - q_2^2} \qquad \dots (9)$$

For example, we calculate $[\alpha]_{q_1,q_2}$, for different values of q_1, q_2 and displayed in Table 1.

Table 1 q_1, q_2**-Deformed Number with Correspondi- Ng Numerical Values**

$[\alpha]_{q_1,q_2}$	Corresponding numerical value when		
	$q_1 = 0.5$, $q_2 = 1.5$	$q_1 = 0.5$, $q_2 = 2.0$	$q_2 = \frac{1}{q_1} = 1.618$
$[0]_{q_1,q_2}$	0.0000	0.0000	0.0000
$[1]_{q_1,q_2}$	1.0000	1.0000	1.0000
$[2]_{q_1,q_2}$	2.2525	4.2500	2.9999
$[3]_{q_1,q_2}$	5.0681	17.0625	7.9994
$[4]_{q_1,q_2}$	11.4033	68.2656	20.9976
$[5]_{q_1,q_2}$	25.6574	273.0664	54.9915
$[6]_{q_1,q_2}$	57.7292	1092.2666	143.9718
$[7]_{q_1,q_2}$	129.8906	4369.0666	376.9104
$[8]_{q_1,q_2}$	292.2540	17476.2666	986.7239
$[9]_{q_1,q_2}$	657.5715	69905.0666	2583.1687
$[10]_{q_1,q_2}$	1479.5358	279620.2666	6762.5396

Now,
bosonic as well as fermionic oscillator algebra

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$$[\alpha]_{q_1,q_2}! = [1]_{q_1,q_2} \times [2]_{q_1,q_2} \times \ldots \times [\alpha]_{q_1,q_2} . \quad \ldots (10)$$

And, the q_1, q_2 −deformed addition law is defined as

$$[\alpha+\beta]_{q_1,q_2} = [\beta]_{q_1,q_2}[\alpha+1]_{q_1,q_2} + [\alpha]_{q_1,q_2}[\beta+1]_{q_1,q_2} - (q_1^2+q_2^2) . [\alpha]_{q_1,q_2}[\beta]_{q_1,q_2} \quad \ldots (11)$$

In accordance with (9), first few generalized Fibonacci basic integers can be written as given in second column of Table 2. Setting, $q_1^2 + q_2^2 = \mu$, and $\sigma = -q_1^2 q_2^2$, generalized Fibonacci basic integers can also be written as displayed in third column of Table 2. Interestingly, for $\mu = \sigma = 1$, $[\alpha]_{q_1,q_2}$, reduces to the well-known Fibonacci sequence as (0,1,1,2,3…) as written in column 4 in the same table and hence the calculus is termed as Fibonacci calculus. Here, we consider, $q_1 \neq q_2$, and both the two parameter belongs to $\mathbb{R}^+$.

In addition, the covariant Fibonacci oscillator algebra can be constructed as,

$$B_m B_n = \frac{q_1}{q_2} B_n B_m, m < n,$$
$$B_m B_n^\dagger = q_1 q_2 B_n^\dagger B_m, m \neq n$$
$$\text{and} , B_1 B_1^\dagger - q_1^2 B_1^\dagger B_1 = q_2^{2\hat{N}} \quad \ldots (12)$$

Table 2 q_1, q_2**-Deformed Number with Correspondi- Ng Numerical Values**

$[\alpha]_{q_1,q_2}$	$[\alpha]_{q_1,q_2}$ q_1, q_2	$[\alpha]_{q_1,q_2}$ μ, σ	$\mu = \sigma = 1$
$[0]_{q_1,q_2}$	0	0	0
$[1]_{q_1,q_2}$	1	1	1
$[2]_{q_1,q_2}$	$q_1^2 + q_2^2$	μ	1
$[3]_{q_1,q_2}$	$q_1^4 + q_1^2 q_2^2 + q_2^4$	$\mu^2 + \sigma$	2
$[4]_{q_1,q_2}$	$q_1^6 + q_1^4 q_2^2 + q_1^2 q_2^4 + q_2^6$	$\mu^2 + 2\,\mu\sigma$	3

But, if we consider the limit, $q_1 = \sqrt{q}$, and $q_2 = 1$, we obtain single parameter AC-type bosonic q -oscillator whereas for the limit $q_1 = \sqrt{q}$, $q_2 = 1/q_1$ the above formulation reduces to BM type bosonic q-oscillator as mentioned in ref. [17]. Most importantly, with the limit $q_1 = q_2 = 1$, the bosonic Fibonacci oscillator algebra changes the form to undeformed bosonic oscillator algebra.

1.3 Preliminaries of two-parameter Fibonacci Calculus

In this section, we will display all the basic features of two-parameter or q_1, q_2 -deformed Fibonacci calculus which are essential to explore our problem. Considering (8) and (9), the transformation from Fock observables to the configuration space (in 1D) will be written as,

$$A^\dagger \to x, A \to \hat{\mathfrak{D}}_{(q_1,q_2),x} . \quad \ldots (13)$$

Here, $\mathfrak{D}_{(q_1,q_2),x}$ is modified two parameter Fibonacci difference operator that can be represented as,

$$\hat{\mathfrak{D}}_{(q_1,q_2),x} \equiv \frac{q_1^2 - q_2^2}{2\ln(q_1/q_2)}\, \hat{\partial}_{(q_1,q_2),x} = Q(q_1,q_2)\hat{\partial}_{(q_1,q_2),x}. \quad \ldots (14)$$

Where, $Q(q_1,q_2) = \frac{q_1^2 - q_2^2}{2\ln(q_1/q_2)}$, and $\hat{\partial}_{(q_1,q_2),x}$ is the Fibonacci differential operator. For any analytic function $g(x)$, $\hat{\partial}_{(q_1,q_2),x} g(x)$ can be written as,

$$\hat{\partial}_{(q_1,q_2),x} g(x) = \frac{f(q_1^2 x) - f(q_2^2 x)}{(q_1^2 - q_2^2)x} . \quad \ldots (15)$$

$\hat{\mathfrak{D}}_{(q_1,q_2),x}$ also obeys the Leibnitz rule as,

$$\hat{\mathfrak{D}}_{(q_1,q_2),x}[f(x)g(x)] = f(q_1^2 x)[\hat{\mathfrak{D}}_{(q_1,q_2),x} g(x)] + g(q_2^2 x)[\hat{\mathfrak{D}}_{(q_1,q_2),x} f(x)].$$

And $\hat{\mathfrak{D}}_{(q_1,q_2),x}(x^n)$ can be written as,

$$\hat{\mathfrak{D}}_{(q_1,q_2),x}(x^\alpha) = Q(q_1,q_2)[\alpha]_{(q_1,q_2)} x^{\alpha-1}. \quad \ldots (16)$$

Note that, the Taylor expansion of a function $g(x)$ about any point a in this formalism is written as

$$g(x) = g(a) + (x-a)\left[\hat{\mathfrak{D}}_{(q_1,q_2),x} g(x)\right]_{x=a} + \frac{(x-a)^2}{[2]_{q_1,q_2}!}\left[\mathfrak{D}^2_{(q_1,q_2),x} g(x)\right]_{x=a} + \cdots \quad \ldots (17)$$

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The two parameters (q_1, q_2) −deformed exponential function can be written as,

$$e^x_{q_1,q_2} = \sum_{r=0}^{\infty} \frac{x^r}{[r]_{q_1,q_2}!} . \qquad \dots(18)$$

From (18), two parameters $(q_1, q_2)-$ deformed trigonometric function can be introduced by using the relation,

$$e^{ix}_{q_1,q_2} = \mathcal{C}_{q_1,q_2}(x) + i\mathcal{S}_{q_1,q_2}(x) . \qquad \dots(19)$$

Where,

$$\mathcal{C}_{q_1,q_2}(x) = \sum_{r=0}^{\infty}(-1)^n \frac{x^{2r}}{[2r]_{q_1,q_2}!} ,$$

$$\text{and } \mathcal{S}_{q_1,q_2}(x) = \sum_{r=0}^{\infty}(-1)^n \frac{x^{2r+1}}{[2r+1]_{q_1,q_2}!} \qquad \dots(20)$$

More interestingly,

$$\widehat{\mathfrak{D}}_{(q_1,q_2),x}(e^{\pm kx}_{q_1,q_2}) = \pm k(e^{\pm kx}_{q_1,q_2}) \qquad \dots(21)$$

And,

$$\int_0^l (e^{\pm kx}_{q_1,q_2}) d_{q_1,q_2}x = \frac{1}{\pm k}\left[(e^{\pm kl}_{q_1,q_2} - 1\right] \qquad \dots(22)$$

In (21), (22) k is a constant. Remarkable that, any function $f(x)$ in such a formalism will be periodic if,

$$f(q_1^2 x) - f(q_2^2 x) = 0$$
$$\Rightarrow f(q_1^2 x) = f(q_2^2 x) . \qquad \dots(23)$$

But it is easy to understand that for $q_1 = \xi$, $q_2 = -1/\xi$, this function will be exactly q-periodic. In such a case,

$$\mathcal{S}_{\xi,-1/\xi}(x) = sin[(\pi/ln\xi)ln|x|],$$

with the period ξ^n, for $n = 0,1,2, \dots$

There are basically three types of infinite potential box problem in quantum mechanics. Here, for simplicity, we consider asymmetric square well problem and suppose a particle of mass m is confined to move inside an infinitely deep well with the following potential

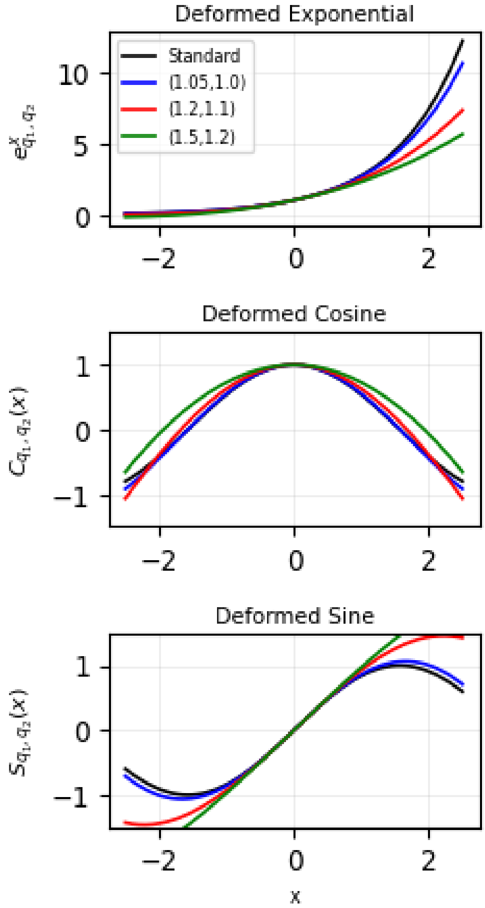

Figure 1 Plot of Two Parameter Deformed Exponential, Sine and Cosine Function Using Python. Here the Functions Are Plotted for Different Values Of q_1, q_2

2. Quantum Mechanics In (q_1, q_2)-Deformed Framework

In (q_1, q_2) −deformed framework, the momentum, position operator and their commutation relation are found to be

$$\hat{\mathcal{P}} = -i\hbar\widehat{\mathfrak{D}}_{(q_1,q_2),x}, \hat{x} = x,$$
$$\text{and } [\hat{x}, \hat{\mathcal{P}}] = i\hbar Q(q_1, q_2). \qquad \dots(24)$$

Then, the $(q_1, q_2)-$deformed uncertainty relation will be,

$$\Delta\hat{x}\Delta\hat{\mathcal{P}} = Q(q_1, q_2)\frac{\hbar}{2} \qquad (25)$$

Accordingly, $(q_1, q_2)-$ deformed time-dependent Schrödinger equation in terms of Hamiltonian and Energy operator will be,

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$$\widehat{\mathcal{H}}_{q_1,q_2}\Phi_{q_1,q_2}(x,t)=\hat{E}\Phi_{q_1,q_2}(x,t) \qquad (26)$$

Using $\hat{E}=i\hbar\frac{\partial\Phi_{q_1,q_2}}{\partial t}$ in (26), we obtain,

$$\left[-\frac{\hbar^2}{2m}\widehat{\mathfrak{D}}^2_{(q_1,q_2),x}+V(x)\right]\Phi_{q_1,q_2}(x,t)=i\hbar\frac{\partial\Phi_{q_1,q_2}}{\partial t} \qquad (27)$$

Here, $\Phi_{q_1,q_2}(x,t)$ is $(q_1,q_2)-$ deformed wave function. Moreover, the scalar product in deformed Hilbert space is defined as

$$\langle\Phi|\Psi\rangle_{q_1,q_2}=\int_{-[\infty]_{q_1,q_2}}^{+[\infty]_{q_1,q_2}}\Phi(x,t)\Psi(x,t)d_{q_1,q_2}x,$$

$$=\int_{-0}^{+\infty}\Phi(x,t)\Psi(x,t)d_{q_1,q_2}x. \qquad (28)$$

And the expectation value of any (q_1,q_2) $-$deformed operator $\hat{O}$ will be,

$$\langle\hat{O}\rangle=\int_{-\infty}^{+\infty}\Phi(x,t)\,\hat{O}\Psi(x,t)d_{q_1,q_2}x, \qquad (29)$$

3. Infinite Potential Well Problem in Fibonacci Calculus

There are basically three types of infinite potential box problem in quantum mechanics. Here, for simplicity, we consider asymmetric square well problem and suppose a particle of mass m is confined to move inside an infinitely deep well with the following potential

$$V(x)=\begin{cases}+\infty, & x<0\\ 0, & 0\le x\le l\\ +\infty, & x>l\end{cases} \qquad (30)$$

In order to study this problem in view of $(q_1,q_2)-$ deformed calculus, we consider the Schr ö dinger equation as written in (26) and rearranging, we obtain,

$$\left(\widehat{\mathfrak{D}}^2_{(q_1,q_2),x}+\kappa^2\right)\Phi_{q_1,q_2}(x)=0 \qquad \ldots(31)$$

Here, $\kappa=\sqrt{2mV/\hbar^2}$. Now, Considering,

$$\Phi_{q_1,q_2}(x)=\sum_{\alpha=0}^{\infty}b_\alpha x^\alpha\ , \qquad \ldots(32)$$

and using (16), we can easily find out $\widehat{\mathfrak{D}}^2_{(q_1,q_2),x}\Phi_{q_1,q_2}(x)$ using (32) . Then after substituting $\widehat{\mathfrak{D}}^2_{(q_1,q_2),x}\Phi_{q_1,q_2}(x)$, $\Phi_{q_1,q_2}(x)$ in (31) and considering $,[0]_{q_1,q_2}=0,[1]_{q_1,q_2}=1,$ we obtain,

$$Q^2\sum_{\alpha=0}^{\infty}b_{\alpha+2}[\alpha+2]_{q_1,q_2}[\alpha+1]_{q_1,q_2}x^\alpha=$$

$$-\kappa^2\sum_{\alpha=0}^{\infty}b_\alpha x^\alpha \qquad \ldots(33)$$

Hence, setting $\kappa^2/Q^2=\hbar\!\!\!k^2$, and comparing both sides of (33) for different powers of x^α, we found, $\Phi_{q_1,q_2}(x)$ can be written as sum of two infinite series where b_0, b_1 are to be determined from proper boundary conditions.

$$\Phi_{q_1,q_2}(x)=b_0\sum_{n=0}^{\infty}(-1)^n\frac{(\hbar\!\!\!k x)^{2n}}{[2n]_{q_1,q_2}!}$$

$$+\,b_1\sum_{n=0}^{\infty}(-1)^n\frac{(\hbar\!\!\!k x)^{2n+1}}{[2n+1]_{q_1,q_2}!}\,. \qquad \ldots(34)$$

Hence,

$$\Phi_{q_1,q_2}(x)=b_0\mathcal{C}_{q_1,q_2}(\hbar\!\!\!k x)+b_1S_{q_1,q_2}(\hbar\!\!\!k x) \quad \ldots(35)$$

Now from the boundary condition $\Phi_{q_1,q_2}(x=0)=0$, we can easily write $S_{q_1,q_2}(\hbar\!\!\!k x)=0$ because either from (36) at $x=0$, $S_{q_1,q_2}(\hbar\!\!\!k x)=0$ unambiguously or from Figure 1 it is clear that $S_{q_1,q_2}(\hbar\!\!\!k x)=0$ at $x=0$. Thus, $b_0=0$ and the wave function $\Phi_{q_1,q_2}(x)$ will be,

$$\Phi_{q_1,q_2}(x)=b_1S_{q_1,q_2}(\hbar\!\!\!k x) \qquad \ldots(36)$$

3.1 Numerical Analysis

Now, as $S_{q_1,q_2}(\hbar\!\!\!k x)$ is an infinite series, it is impossible to find out any periodicity within it as well as it is also difficult to find proper numerical interpretation. In order to discuss it numerically, we suppose a limiting condition $q_1=\xi$, $q_2=-1/\xi$. Hence, the wave function can be written as,

$$\Phi_{q_1,q_2}(x)=b_1S_{\xi,-1/\xi}(\hbar\!\!\!k x)=\ sin\left[\frac{\pi}{\ln\xi}ln|x|\right]. \qquad \ldots(37)$$

And the normalized wave function will be

$$\Phi_{q_1,q_2}(x)=\sqrt{\frac{2}{l}\left[\frac{1}{1-f(\hbar\!\!\!k l)}\right]}\ sin\left[\frac{\pi}{\ln\xi}ln|x|\right]. \qquad \ldots(38)$$

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Where, $f(\hbar l) = \left[\frac{\{v \sin(vln|\hbar l|)+cos(vln|\hbar l|)\}}{1+(v)^2}\right]$, and $v = \frac{2\pi}{\ln\xi}$, In addition, considering the properties of wave function and remembering the periodicity of $(\xi, 1/\xi)-$deformed $S_{q,1/q}$, we obtain the energy eigen value as,

$$E_{\xi,n} = \frac{\hbar^2}{2ml^2}\left(\frac{\xi^2 - 1/\xi^2}{4\ln(\xi)}\right)^4 \xi^{2n} \qquad (39)$$

It is different from the conventional expression $E_n = \frac{\hbar^2}{2ml^2}n^2$.

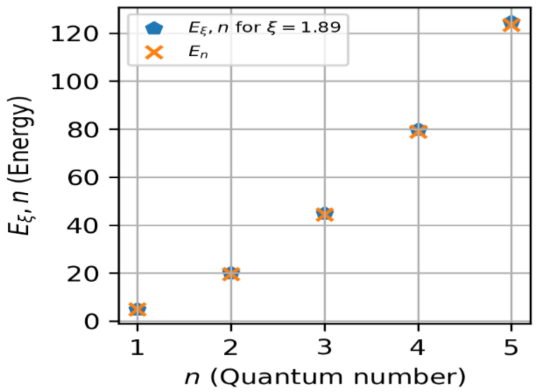

Figure 2 Plot of $E_{\xi,n}$ and $E_n$for $n = 1$ to 5

In Figure 2, we consider, $m = \hbar = 1$ and the energy of deformed levels is dependent on both n , and ξ. All the blue pentagonal points are indicating the energy of the concerned levels (with ξ=1.89) which are very closer to cross points. From Figure 3, it is clear that the deformation is more in higher energy levels.

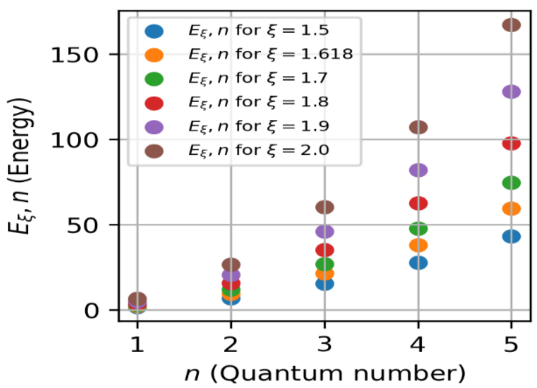

Figure 3 Plot of $E_{\xi,n} = \frac{\hbar^2}{2ml^2}\left(\frac{\xi^2-1/\xi^2}{4\ln(\xi)}\right)^4 \xi^{2n}$ for different values of ξ

But, if we consider $q_1 = \sqrt{\xi},\ q_2 = -1/q_1$,then

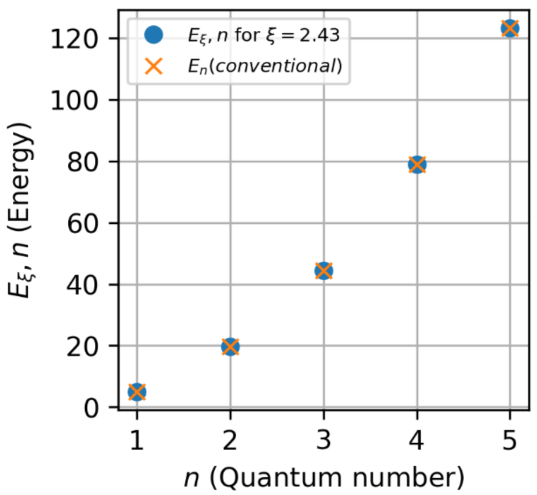

Figure 4 Plot of $E_{\xi,n} = \frac{\hbar^2}{2ml^2}\left(\frac{\xi-1/\xi}{2\ln(\xi)}\right)^4 \xi^{2n}$ for $\xi = 2.43$ and $E(conventional) = \frac{\hbar^2}{2ml^2}n^2$for energy up to fifth energy levels

From Figure 4 it is clear that, at ξ=2.43, the deformed energy values become nearly equal to the conventional energy values for each level. It is also to be noticed that for larger values of ξ, energy associated with different energy levels are greater which indicates a change in the mass value. Basically, it is supposed to be the effective mass of the particle confined in the infinite potential box as indicated analytically in ref. [12]. Thus, here ξ can be treated as a fitment parameter for calculating effective mass of any particle confined in an asymmetric infinite potential box.

Conclusion

In last few years, a number of deformed schemes have been introduced to understand conventional quantum mechanics, non-extensive statistical mechanics, and condensed matter physics. Fibonacci anyons, a type of quasiparticle present in 2-D materials can also be studied theoretically with deformed q-calculus. It is important to note that very recently, in ref. [16], it is claimed that anyons may be used to encode and manipulate information securely

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with the use of quantum gates which is useful to explore much more about intermediate-scale quantum processors. On the basis of two parameter Fibonacci calculus or q_1, q_2-deformed calculus, at first, we have solved Schrödinger equation which describes the position of a particle inside an asymmetric infinite quantum well. It is observed that within the limit $q_1 \to 1, q_2 \to 1$, all the obtained results turned to their conventional form as expected. Then, considering two special situations, $q_1 = \xi$, $q_2 = -1/\xi$, $q_1 = \sqrt{\xi}$, $q_2 = -1/q_1$ and using the periodicity of q_1, q_2-deformed sine and cosine functions, we graphically plot the deformed energy with quantum numbers. It is observed that, the deformation of energy values is larger for higher quantum states and for any particular energy level this deformation is may be due to the varying effective mass of the confined particle which is a matter of further investigation. However, this formalism may be useful to study a number of potential problems of quantum mechanics to explore more insightful observations. Moreover, this can add a good pedagogical example of contemporary mathematical physics as well as theoretical chemistry.

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References

[1]. Kac, V. G., & Cheug, P. (2002). Quantum Calculus. Springer-verlag, NY. https://link . springer.com/book/10.1007/978-1-4613-0071-7

[2]. Trjitzinsky, W.J. Analytic theory of linearq-difference equations. Acta Math. **61**, 1–38 (1933). https://doi.org/10.1007/BF02547785

[3]. Earnst, T. (2003). A Method for q-calculus. J. nonlinear math. Phys. A, 10, 487-525. https://doi.org/10.2991/jnmp.2003.10.4.5

[4]. Alanazi, A. M. et. al, (2020). The falling body problem in quantum calculus, Frontiers in Physics, 8 (Art.43) 1-5 https://doi.org/10.3389/fphy.2020.00043

[5]. Bhattacharya, P. (2024). Features of Projectile Motion in Quantum Calculus, Mapana-Journal of Sciences, 23(1), 35-45. https://doi.org/10.12723/mjs.68.2

[6]. Bhattacharya, P. et. al. (2024). The Exact Solution to Wave Equation Using RqDTM in Quantum Calculus. AIP Conf. Proc. 3149, 140025. https://doi.org/10.1063/5.0225273

[7]. Bhattacharya, P. et. al. (2024). Study of Damped Harmonic Oscillator in View of Quantum Calculus. AIP Conf. Proc. 3149, 140028. https://doi.org/10.1063/5.0224388

[8]. Bhattacharya, P. (2024) Simple Harmonic Motion in View of Quantum Calculus, Advances in Mathematical and Computational Sciences, Proceedings of the ICRTMPCS International Conference 2023 De Gruyter, Germany, (ISBN: 978-3-11-130437-3). https://doi.org/10.1515/9783111313634-034

[9]. Bhattacharya, P. (2023) Solution to Laplace's Equation Using Quantum Calculus, International Journal of Engineering Technology and Management Sciences,7(5), 522-531. DOI:10.46647/ijetms.2023.v07i05.066

[10]. Minahan, J A. "The q-Schroedinger equation." Modern Physics Letters A; (United States), vol. 5:31, Dec. 1990. https://doi.org/10.1142/S021773239000305X

[11]. Ali, K. K., Shehata, A. S., Mohamed, M. S., Alqurashi, T. J., & Maneea, M. (2025). An innovative q-homotopy analysis approach for solving generalized q-fractional non-linear biological population model. AIP Advances, 15(2).

[12]. Chung, W. S. & Hassanabadi, H. (2020). Fermi Energy in the q-deformed quantum Mechanics, Modern Physics Letter A, 2050074. Doi:10.1142/S0217732320500741.

[13]. Chung, W. S. & Hassanabadi, H. (2019). q-deformed quantum Mechanics based on q-addition, Fortschritte der Physik, 67(4). DOI:10.1002/prop.201800111.

[14]. Micu, M. (1999). A q-deformed Schrodinger Equation. J. Phys. A: Math. Gen. 32, 7765-7777. DOI 10.1088/0305-4470/32/44/312
[15]. Lavanjo, A. & Gervino, G. (2009). Quantum Mechanics in q-deformed Calculus, journal of physics: Conference Series, 174 012071. Doi:10.1088/1742-6596/174/1/012071
[16]. Shibo, Xu et. al. (2024) Non-Abelian braiding of Fibonacci anyons with a superconducting processor, Nature Physics, 20,1469–1475. https://doi.org/10.1038/s41567-024-02529-6.
[17]. Algin, A & Chung, W. S. (2024). Two-parameter deformed quantum Mechanics based on Fibonacci Calculus and Debye Crystal Model of Two Parameter Deformed Quantum Mechanics, Eur. Phys. J. Plus. 139:198. https://doi.org/10.1140/epjp/s13360-024-05008-2.
[18]. Pashaev, K. O. (2020) Quantum Calculus of Fibonacci Divisors and Infinite Hierarchy of Bosonic Fermionic Golden Quantum Oscillators, http://arxiv.org/abs/2010.12386v1.
[19]. Pashaev, K. O. & Sengul. (2011) Golden Quantum Oscillator and Binet-Fibonacci Calculus. Journal of Physics A: Mathematical and Theoretical. doi: 10.1088/1751-8113/45/1/015303.