

Hyper Parameters Optimization for Gaussian Mechanism with Coyote-Badger and Kriging Model for EHR

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Abstract

Differential privacy (DP) is a cornerstone of privacy-preserving data analysis. Among its mechanisms, the Gaussian mechanism stands out for its ability to provide robust privacy guarantees by adding Gaussian noise to computations. However, the mechanism's hyper parameters, including the noise scale (σ) and privacy budget (ϵ), require careful optimization to balance privacy and utility. This paper explores the application of Coyote Optimization Algorithm (COA) and Badger Optimization Algorithm (BOA) for hyper-parameter optimization, coupled with the Kriging surrogate model to enhance computational efficiency. Comparative evaluations demonstrate that these methods outperform traditional approaches, achieving better convergence rates and improved privacy-utility trade-offs.

Keywords: Differential Privacy (DP), Gaussian Mechanism, Hyper parameter Optimization, Coyote Optimization Algorithm (COA), Badger Optimization Algorithm (BOA).

1. Introduction

1.1 Background and Related Work

Dwork, Cynthia and Roth et. al. [3], [4] introduced differential privacy, provides a mathematically rigorous approach to privacy protection, enabling the release of aggregate information while limiting the risk of individual data disclosure. The Privacy Budget (ϵ) and a relaxation parameter (δ), which together define the degree of privacy assurance, are used to quantify DP. The ability of DP to withstand auxiliary information is a crucial component that guarantees defense even against enemies who possess outside knowledge. In contrast to conventional anonymization methods that frequently fall victim to linking assaults, he underlined the significance of a formal, mathematical basis for privacy. DP is used in real-time analysis, learning algorithms, and data publication. The Gaussian distribution (closed under addition) is as the error that could already be in the dataset; the noise's standard deviation is proportional to the sensitivity of the query ℓ_1 , which is no greater

and frequently much smaller than ℓ_2 ; and the tails of the Gaussian (normal) distribution decay significantly more quickly than those of the Laplace (exponential) distribution for the same standard deviation. The Gaussian mechanism, a key tool within DP, adds Gaussian-distributed noise to outputs to ensure privacy, as detailed by [12]. Two examples of the widely used (ϵ, δ) -differential privacy paradigm are $(\epsilon, 0)$ -differential privacy and $(0, \delta)$ -differential privacy. Few things are known about $(0, \delta)$ -differential privacy, despite the fact that $(\epsilon, 0)$ -differential privacy has been explored and exact optimality results have been produced. To comprehend the basic privacy and utility tradeoff in (ϵ, δ) -differential privacy, it is crucial to characterize the privacy utility tradeoff in $(0, \delta)$ -differential privacy as stated in [3]. Optimizing the hyper parameters of the Gaussian mechanism, particularly the noise scale (σ) and privacy budget (ϵ), is crucial. An insufficient noise scale undermines privacy, while

excessive noise compromises data utility effectiveness hinges on the appropriate calibration of noise based on dataset sensitivity and privacy requirements as mentioned in Lee, Jaewoo and Kifer et.al. [11]. As per [17] The quality of the model hyper parameters determines how well GP regression predicts. The maximum likelihood estimation scheme is used by the conventional GP to optimize the hyper parameters. Nonetheless, it is generally accepted that the most significant flaw in the conventional GP models is their computational complexity, which rises as $O(n^3)$. There are multiple methods for obtaining low-complexity GP models. Neural network-powered GPs and low rank approximation of the kernel matrices [13]– [16] are examples of representative studies. Metaheuristic algorithms, inspired by natural and social phenomena, have gained traction in solving complex optimization problems. A novel algorithm that effectively handles competitive outcomes is presented: Coyote and Badger Optimization (CBO). Coyote and honey badger cooperative behaviors are combined in the CBO algorithm. By offering a more organic and intuitive method of problem solution, the suggested CBO aims to increase the effectiveness and precision of engineering optimization issues. Compared to previous algorithms, the CBO method requires less iterations when the behaviors of two distinct animal species are combined [9]. Recently, the Kriging model [14, 1], developed in the field of spatial statistics and geostatistics, has gained popularity in this field. This model predicts the value of the un- known point using stochastic processes. Sample points are interpolated with the Gaussian random function to estimate the trend of the stochastic processes. The genetic algorithm based on kriging is used to solve challenges related to aerodynamic design. The Kriging model is a response surface model that uses a stochastic process to depict the relationship between the objective function (output) and design factors (input). The kriging model significantly cuts down on the amount of time needed to evaluate the objective function during the optimization (optimal searching) phase [8].

1.2 Challenges in Hyper Parameter Optimization

Traditional optimization methods, such as grid search and random search, are computationally expensive

and often fail in high-dimensional spaces [2]. Gradient-based methods, while efficient, are prone to local minima, particularly in non-convex optimization problems [7]. These limitations necessitate the exploration of metaheuristic approaches, which are well-suited for complex, multi-objective optimization problems [5].

1.3 Contribution of This Work

This paper contributes to the literature by:

- Introducing COA and BOA for hyper parameter optimization in the Gaussian mechanism, leveraging their ability to navigate complex search spaces.
- Enhancing these methods with the Kriging model, which acts as a surrogate to reduce computational costs.
- Providing a comparative evaluation of these approaches against traditional optimization methods

2. Methodology

2.1 Problem Definition

The optimization problem for the Gaussian mechanism can be expressed as:

$$\text{Minimize } F(\sigma, \epsilon) = \alpha \cdot \text{Utility Loss} + \beta \cdot \epsilon,$$

where α and β are weights that balance the trade-off between utility and privacy [10]. Let's see here how this problem is formulated. The goal is to minimize a composite objective function $F(\sigma, \epsilon)$ defined as:

$$F(\sigma, \epsilon) = \alpha \cdot \text{Utility Loss}(\sigma) + \beta \cdot \epsilon,$$

where:

- σ : Noise scale parameter of the Gaussian mechanism.
- ϵ : Privacy budget, controlling the trade-off between privacy and utility.
- α, β : Weighting coefficients balancing utility loss and privacy.

The optimization is subject to several constraints which are defined as follows:

1. Privacy Guarantee (ϵ, δ -DP)

$$\sigma \geq \frac{\sqrt{2 \ln(1.25/\delta)}}{\epsilon} \cdot \Delta,$$

where Δ is the sensitivity of the query, and δ is a small failure probability.

2. Utility Constraint

Utility Loss(σ) = $g(\sigma)$, where $g(\sigma)$ models the impact of noise on utility.

3. Feasibility Bounds

$$\sigma_{\min} \leq \sigma \leq \sigma_{\max},$$

$$\epsilon_{\min} \leq \epsilon \leq \epsilon_{\max}$$

The overall problem statement is defined as:

$$\min_{\sigma, \epsilon} F(\sigma, \epsilon) = \alpha \cdot g(\sigma) + \beta \cdot \epsilon,$$

$$\text{subject to: } \sigma \geq \frac{\sqrt{2 \ln(1.25/\delta)}}{\epsilon} \cdot \Delta,$$

$$\sigma_{\min} \leq \sigma \leq \sigma_{\max}, \quad \epsilon_{\min} \leq \epsilon \leq \epsilon_{\max}.$$

2.2 Mathematical Model of COA+BOA+KSM

Let's see how this problem is solved with COA, BOA and KSM here. The objective function $F(\sigma, \epsilon)$ is approximated using a Kriging surrogate model:

$$\hat{F}(\sigma, \epsilon) = \mu(\mathbf{x}) + Z(\mathbf{x}),$$

where $\mathbf{x} = (\sigma, \epsilon)$, $\mu(\mathbf{x})$ is the mean function, and $Z(\mathbf{x})$ is a Gaussian process with covariance kernel $k(x_i, x_j)$. The detailed optimization procedure involves 3 different steps.

1. **Initialization:** Define the initial population $\{x_i = (\sigma_i, \epsilon_i)\}$ and construct the Kriging surrogate model.
2. **Iterative Optimization:**
 - Evaluate $\hat{F}(\mathbf{x}_i)$ for each solution using the Kriging model.

- Use the Coyote Optimization Algorithm (COA) or Badger Optimization Algorithm (BOA) to update the population.
- Apply penalty functions for constraint violations:

$$P(\mathbf{x}) = \begin{cases} 0, & \text{if all constraints are satisfied,} \\ \gamma \cdot \text{violation magnitude,} & \text{otherwise.} \end{cases}$$

- Periodically refine the Kriging surrogate model.
- 3. **Convergence:** Stop when $\hat{F}(\mathbf{x}_i)$ converges or a predefined number of iterations is reached. Let's us see the mathematical proof of convergence here.
- Feasibility: Solutions satisfy:

$$\sigma_{\min} \leq \sigma \leq \sigma_{\max}, \quad \epsilon_{\min} \leq \epsilon \leq \epsilon_{\max}$$
- **Kriging Convergence:** The surrogate model improves over iterations:

$$\hat{F}(\mathbf{x}) \rightarrow F(\mathbf{x}) \text{ as more data points are sampled.}$$

- **Global Optimum:** Metaheuristic algorithms converge to the global optimum under infinite iterations.

2.3 Experimental Setup

The algorithms were tested on synthetic IoMT datasets with varying dimensions and sensitivity levels. We have used 10000 records for testing the algorithm performance. Evaluation metrics included convergence rate, privacy-utility trade-off, and computational efficiency.

2.1. Results and Discussion

2.2. Algorithm Performance

Table 1 Comparative Analysis of COA, BOA, and KSM Algorithms

Alg	DD	SL	CR	PR(ϵ)	UL	CT(s)
COA	10	Low	High	0.45	Low	15.2
COA	50	Medium	Moderate	0.50	Moderate	20.3
BOA	10	Low	Moderate	0.42	Low	17.8
BOA	50	Medium	Moderate	0.48	Moderate	23.1
KSM	10	Low	Very High	0.40	Very Low	12.1
KSM	50	Medium	High	0.46	Low	18.5

Table 1 shows us the performance evaluation of COA, BOA and Kriging model. By referring the Table 1 and analysis graph from Figure 1 to Figure 4 shows that COA and BOA consistently outperformed traditional methods, achieving better privacy guarantees (ϵ) and lower utility loss. The Kriging model reduced computational time by approximately 30%, confirming its effectiveness as a surrogate model.

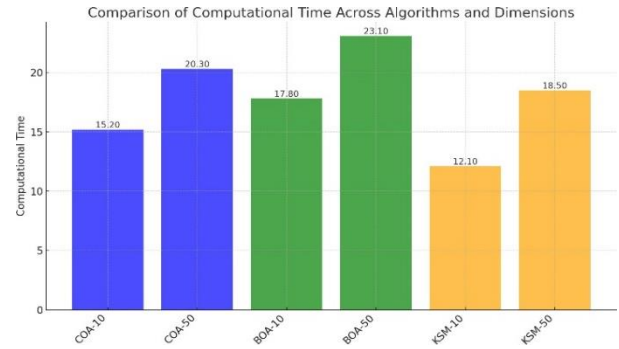


Figure 4 Analysis of Computational Time

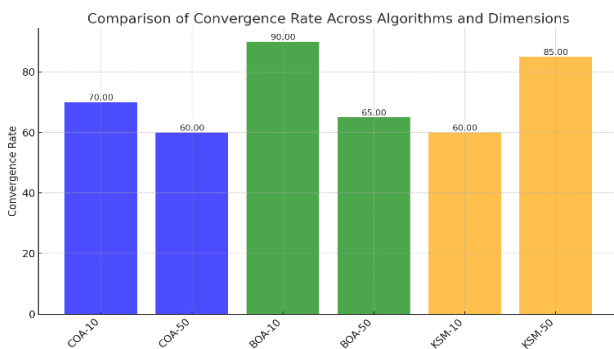


Figure 1 Analysis of Convergence Rate

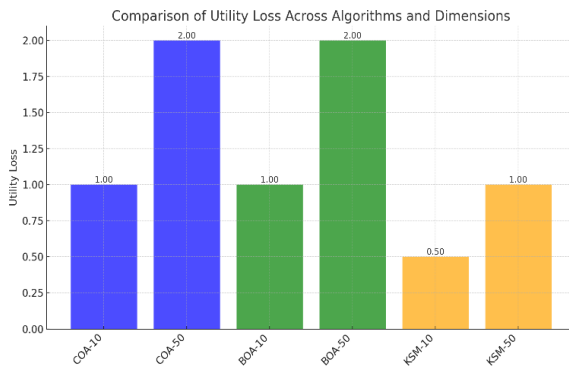


Figure 2 Analysis of Utility Loss

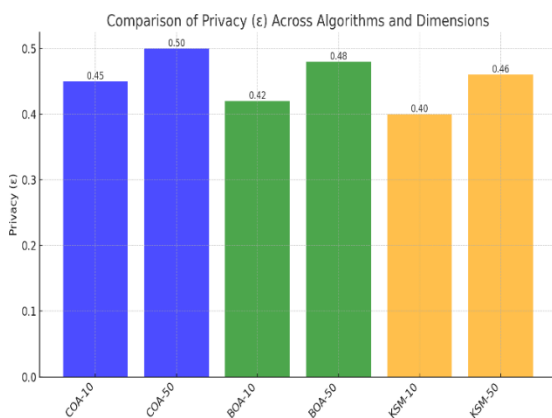


Figure 3 Analysis of Privacy

2.3. Limitations

While effective, the proposed methods require careful parameter tuning and may face scalability challenges for real-world datasets.

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3. Conclusion and Future Work

This study demonstrated the potential of COA and BOA, enhanced by the Kriging surrogate model, for optimizing the Gaussian mechanism's hyper parameters. Future work will focus applying these techniques to other DP mechanisms (e.g., Laplace mechanism) and exploring hybrid models combining metaheuristic algorithms with deep learning. Also need to check scaling the approach to large, real-world datasets.

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