# Synchronization of Complex System by Designing a Controller

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## Abstract

Synchronization of two complex identical systems using nonlinear control is presented by this paper. For emphasis on the chaos behavior, initial conditions sensitivity and Lyapunov exponent is calculated. For the synchronization of the complex master and slave system, the nonlinear control method is proposed effectively.

**Keywords:** Complex Identical System, Numerical Simulation, Nonlinear Control, Synchronization

## 1. Introduction

After the pioneering work on chaos control [1-2], synchronization has attracted wide attention. Generally, two systems are used in synchronization one master as an input system and the other slave as an output System. Synchronization has become a very active area of interest in nonlinear science and also in the area of applied mathematics and automation engineering [7-13]. Many more applications as secure communication [3-6] the topic of synchronization has various applications. Different effective methods are used for the different chaotic systems which are based on the different methods recently back stepping and active control methods have gained popularity in the area of synchronization as these methods are more powerful and effective. Through the transmission of the signal, the trajectory of the slave system approaches asymptotically to the trajectory of the master system which is the input system so that the error dynamics converge to zero [14-16]. When several single oscillators are coupled together then a complicated system is obtained. For the study of these types of oscillators complex variables are used which are more convenient. Based on the Lyapunov function for the determination of the controllers the nonlinear control technique is used and also for synchronizing two identical complex chaotic systems. The paper is planned as: In section I1 Design of nonlinear control method. In section III, System description and nonlinear control method are used for the synchronization of two identical complex systems. Simulation agreements are shown in section IV which presents the proposed system as efficient. Lyapunov exponents of the dynamical system are shown in Figure 1.

### 1.2 Design of a controller system

By using the relation consider the master system

$\dot{x_1} = A_1 x_1 + g_1(x_1)$

Where $x_1$ (t) is the state vector f n- dimensionof the master system, $A_1 \in R^{n*n}$ is the system parameters matr , and the function $g_1(x_1): R^n \to R^n$ is a nonlinear function. The slave system is defined as by adding the control input vector

$\dot{x_2} = A_2 x_2 + g_2(x_2) + u$

Where $x_2$ (t) is the state vector of n- dimension of the slave system, the slave system parameters matrix is $A_2 \in R^{n*n}$, and the function $g_2(x_2): R^n \to R^n$ is a nonlinear function.when the two chaotic systems are identical the $A_1 = A_2$ and $g_1(x_1) = g_2(x_2)$ but when the system is not identical then $A_1 \neq A_2$ and also $g_1(x_1) \neq g_2(x_2)$

By using the master and slave equations consider the error dynamical system

$\dot{e} = A_2 x_2 + g_2(x_2) + u - (A_1 x_1 + g_1(x_1))$, where $e = x_2 - x_1$ is the error state vector. The considered problem is to design an appropriate controller $u$ thet rajectory of the

slave system asymptotically approaches to the master system. When error vector is converges to zero as time goes to infinity the synchronization is obtained. Consider the error function in terms of Lyapunov

$$v(\dot{e}) = \frac{1}{2}e^t e$$

Here the positive definite function is $v(\dot{e})$. For obtaining the synchronization the controller u is selected in such a way that $v(\dot{e}) < 0$, then. Assume that the states of input and output system are measurable and parameters are known.

## 2. System Description and Synchronization of Two Identical Systems

The Rossler system [15] is given by Rossler for verification of effectiveness of chaos control technique The coupled non-linear differential equation is

$$\dot{x}_1 = \alpha(x_2 - x_1)$$
$$\dot{x}_2 = (\Upsilon - \alpha)x_1 - x_1x_3 + \Upsilon x_2$$
$$\dot{x}_3 = -\beta x_3 - \delta x_4 + x_1x_2$$
$$\dot{x}_4 = -dx_4 + fx_3 + x_1x_2$$

Now replace the real variable of the system $(x_1, x_2, x_3, x_4)$ by the complex variables

$$x_1 = X_1 + \mathbf{i}\, X_2$$
$$x_2 = X_3 + \mathbf{i}\, X_4$$
$$x_3 = X_5 + \mathbf{i}\, X_6$$
$$x_4 = X_7 + \mathbf{i}\, X_8 \qquad (1.1)$$

We will get the following complex system

$$\dot{x}_1 = \alpha(x_3 - x_1)$$
$$\dot{x}_2 = \alpha(x_4 - x_2)$$
$$\dot{x}_3 = (\Upsilon - \alpha)x_1 - x_1x_5 + \Upsilon x_3$$
$$\dot{x}_4 = (\Upsilon - \alpha)x_2 - x_2x_6 + \Upsilon x_4$$
$$\dot{x}_5 = -\beta x_5 - \delta x_7 + x_1x_3$$
$$\dot{x}_6 = -\beta x_6 - \delta x_8 + x_2x_4$$
$$\dot{x}_7 = -dx_7 + fx_5 + x_1x_3$$
$$\dot{x}_8 = -dx_8 + fx_6 + x_2x_4 \qquad (1.2)$$

System (1.2) is considered a master system and the matrix for the system i

$$A = \begin{bmatrix} -a & 0 & a & 0 & 0 & 0 & 0 & 0 \\ 0 & -a & 0 & a & 0 & 0 & 0 & 0 \\ \gamma - a - x_5 & 0 & \gamma & 0 & -x_1 & 0 & 0 & 0 \\ 0 & \gamma - a - x_6 & 0 & \gamma & 0 & -x_2 & 0 & 0 \\ x_3 & 0 & x_1 & 0 & -\beta & 0 & -\delta & 0 \\ 0 & x_4 & 0 & x_2 & 0 & -\beta & 0 & -\delta \\ x_3 & 0 & x_1 & 0 & f & 0 & -d & 0 \\ 0 & x_4 & 0 & x_2 & 0 & f & 0 & -d \end{bmatrix}$$

The slave that is the output system is

$$\dot{y}_1 = \alpha(y_3 - y_1) + u_1$$
$$\dot{y}_2 = \alpha(y_4 - y_2) + u_2$$
$$\dot{y}_3 = (\Upsilon - \alpha)y_1 - y_1y_5 + \Upsilon y_3 + u_3$$
$$\dot{y}_4 = (\Upsilon - \alpha)y_2 - y_2y_6 + \Upsilon y_4 + u_4$$
$$\dot{y}_5 = -\beta y_5 - \delta y_7 + y_1y_3 + u_5$$
$$\dot{y}_6 = -\beta y_6 - \delta y_8 + y_2y_4 + u_6$$
$$\dot{y}_7 = -dy_7 + fy_5 + y_1y_3 + u_7$$
$$\dot{y}_8 = -dy_8 + fy_6 + y_2y_4 + u_8 \qquad (1.3)$$

Where $u = [u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8]^T$ is the control input vector to be determined and $T$ is the transpose. Trajectory of slave system asymptotically approaches to that of master system, we wish to estimate appropriate nonlinear controller $u_i$ where $i = 1,2,3, \dots.8$. Now consider the error dynamical system is

$$\dot{e}_{i=} y_i - x_i \qquad \text{i=1, 2, .8} \qquad (1.4)$$
$$\dot{e}_{i=} y_1 - x_1$$
$$\dot{e}_{i=} \alpha(y_3 - y_1) + u_1 - [\alpha(x_3 - x_1)]$$
$$\dot{e}_{i=} \alpha(e_3 - e_1) + u_1$$
$$\dot{e}_{2=} y_2 - x_2$$
$$\dot{e}_{2=} \alpha(y_4 - y_2) + u_2 - [\alpha(x_4 - x_2)]$$
$$\dot{e}_{2=} \alpha(e_4 - e_2) + u_2$$
$$\dot{e}_{3=} y_3 - x_3$$
$$\dot{e}_{3=} (\Upsilon - \alpha)y_1 - y_1y_5 + \Upsilon y_3 + u_3 - [(\Upsilon - \alpha)x_1 - x_1x_5 + \Upsilon x_3]$$
$$\dot{e}_{3=} (\Upsilon - \alpha)e_1 - y_1y_5 + \Upsilon e_3 + u_3 - x_1x_5$$
$$\dot{e}_{4=} y_4 - x_4$$
$$\dot{e}_{4=} (\Upsilon - \alpha)y_2 - y_2y_6 + \Upsilon y_4 + u_4 - [(\Upsilon - \alpha)x_2 - x_2x_6 + \Upsilon x_4]$$
$$\dot{e}_{4=} (\Upsilon - \alpha)e_2 - y_2y_6 + \Upsilon e_4 + u_4 - x_2x_6$$
$$\dot{e}_{5=} y_5 - x_5$$
$$\dot{e}_{5=} -\beta y_5 - \delta y_7 + y_1y_3 + u_5 - [-\beta x_5 - \delta x_7 + x_1x_3]$$
$$\dot{e}_{6=} y_6 - x_6$$
$$\dot{e}_{6=} -\beta y_6 - \delta y_8 + y_2y_4 + u_6 - [-\beta x_6 - \delta x_8 + x_2x_4]$$
$$\dot{e}_{6=} -\beta e_6 - \delta y_8 + y_2y_4 + u_6 - x_2x_4$$
$$\dot{e}_{7=} y_7 - x_7$$
$$\dot{e}_{7=} -dy_7 + fy_5 + y_1y_3 + u_7 - [-dx_7 + fx_5 + x_1x_3]$$
$$\dot{e}_{7=} de_7 + fe_5 + y_1y_3 + u_7 - x_1x_3$$
$$\dot{e}_{8=} y_8 - x_8 -$$
$$\dot{e}_{8=} -dy_8 + fy_6 + y_2y_4 + u_8 - [-dx_8 + fx_6 + x_2x_4]$$
$$\dot{e}_{8=} -de_8 + fe_6 + y_2y_4 + u_8 - x_2x_4 \qquad (1.5)$$

This error system can be considered as control problem with the control input vector $u_i$ Consider the Lyapunov function

$$v\dot{(e)} = \frac{1}{2}e^t e \qquad (1.6)$$

In order to make (1.6) negative, the controllers $u_i$ are

$$u_1 = 2\alpha e_1 - \alpha e_3$$
$$u_2 = 2\alpha e_2 - a e_4$$
$$u_3 = -[\gamma - \alpha]e_1 + y_1 y_5 + x_1 x_5 + (1 - \gamma)e_3$$
$$u_4 = -[\gamma - \alpha]e_2 + y_2 y_6 + x_2 x_6 + (1 - \gamma)e_4$$
$$u_5 = \delta e_7 - y_1 y_3 + x_1 x_3 + (1 + \beta)e_5$$
$$u_6 = \delta e_8 - y_2 y_4 + x_2 x_4 + (1 + \beta)e_6$$
$$u_7 = -f e_5 - y_1 y_3 + x_1 x_3 + (1 + d)e_7$$
$$u_8 = -f e_6 - y_2 y_4 + x_2 x_4 + (1 + d)e_8 \qquad (1.7)$$

By using appropriate controllers $u_i$ the equation (1.6) becomes

$$v\dot{(e)} = -\alpha e_1{}^2 - a e_2{}^2 - e_3{}^2 - e_4{}^2 - e_5{}^2 - e_6{}^2 - e_7{}^2 - e_8{}^2 < 0 \qquad (1.8)$$

Since the function $v\dot{(e)}$ is a negative definite function and the error state $\lim_{t \to \infty} \lVert e(t) \rVert = 0$, approaching synchronization of master slave system.

The final slave system by using the equations (1.7) is

$$\dot{y}_1 = \alpha(x_3 - 2x_1 + y_1)$$
$$\dot{y}_2 = \alpha(x_4 - 2x_2 + y_2)$$
$$\dot{y}_3 = \gamma(x_1 + x_3) + x_1 x_5 + y_3 - x_3 + \alpha x_1$$
$$\dot{y}_4 = \gamma(x_2 + x_4) + x_2 x_6 + y_4 - x_4 + \alpha x_2$$
$$\dot{y}_5 = -x_5 - \delta x_7 + x_1 x_3 + y_5$$
$$\dot{y}_6 = -\beta x_6 - \delta x_8 + x_2 x_4 + y_6$$
$$\dot{y}_7 = -d x_7 + f x_5 + x_1 x_3 + y_{7} - x_7$$
$$\dot{y}_8 = -d x_8 + f x_6 + x_2 x_4 + y_8 - x_8 \qquad (1.9)$$

## 3. Numerical Simulation

The initials conditions are $x(0) = (0.1,0,0.1,1,0.1,1,0.1,1)^T$ and $y(0) = (0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1)^T$ of two equations (1.2) and (1.9) and the values of $(\alpha, \beta, \gamma, \delta, d, f)$ as $(10,10,20,10,10,10)$ are selected in such a way that the system is solved numerically By using the MATLAB . Figure 3(a) to 3(d) shows the error dynamics with time t, shows that error system converges to zero and the two system are synchronized. Figure 4 (a) to 4(d) shows the time series of signals between $x_i$ and $y_i$ where $i = 1,2,....8$. Figure 2(a) to 2(e) shows the chaotic behaviour of the system (1.2) for the values of $\alpha, \beta, \gamma, \delta, d, f)$ as $(10,10,\frac{8}{3},10,10,10)$
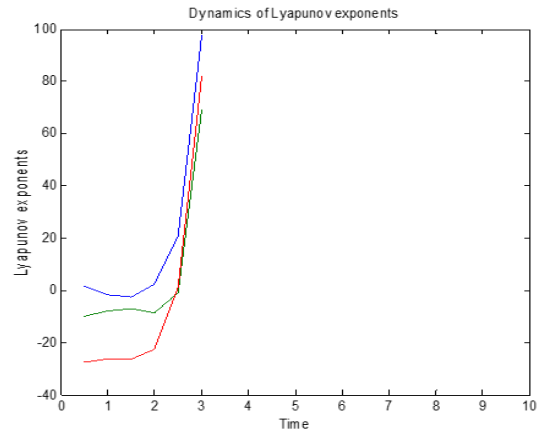


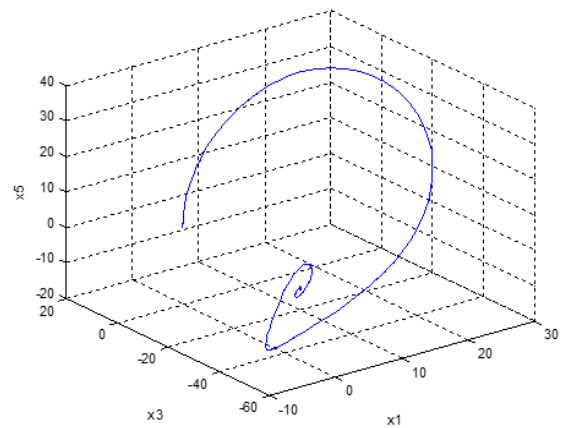**Figure 1** **Lyapunov exponents of dynamical system**



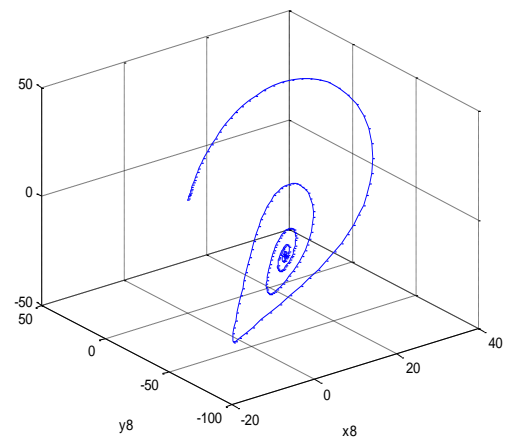**Figure 2(a)** **Chaotic behavior of the system between $x_{1,}x_{3,}x_5$**
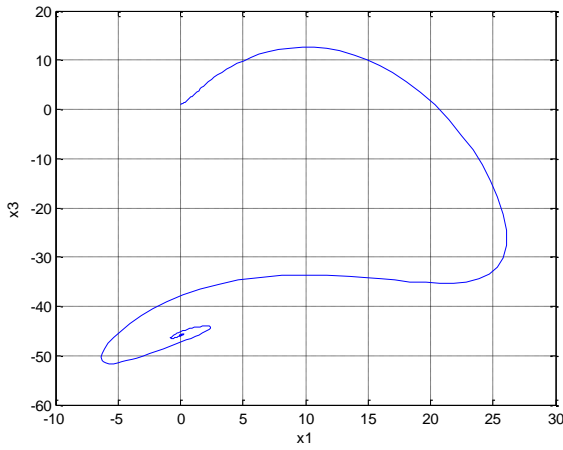


**Figure 2(b)** **The Chaotic nature of the system between $x_{1,}x_{3,}x_8$**

**Figure 2(c)** The chaotic nature between $x_1, x_3$

**Figure 2(d)** The chaotic nature between the $x_1, x_5$
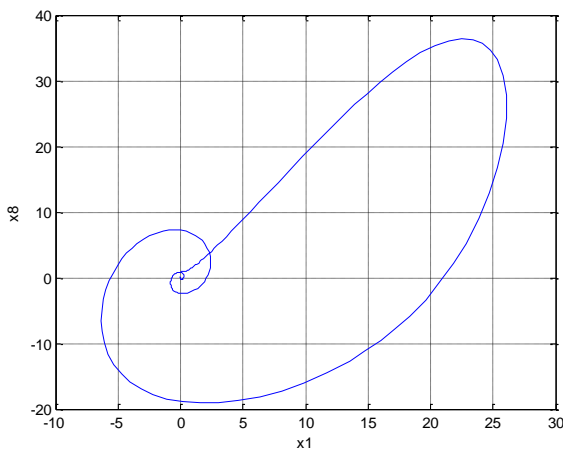
**Figure 2(e)** The chaotic nature between the system $x_1, x_8$

**Figure 3(a)** The Error behavior between $e_1$ and $e_2$

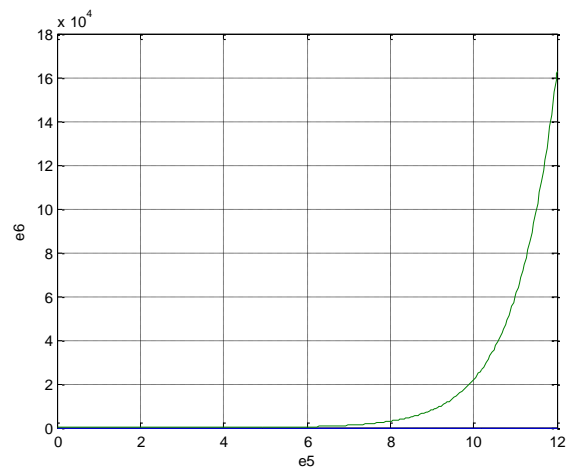**Figure 3(b)** The Error behavior between $e_3$ and $e_4$

**Figure 3(c)** The Error behavior between $e_5$ and $e_6$
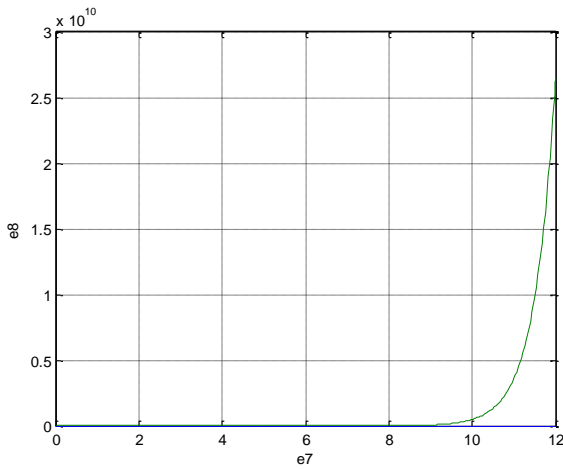
**Figure 3(d)** The Error behavior between $e_7$ and $e_8$
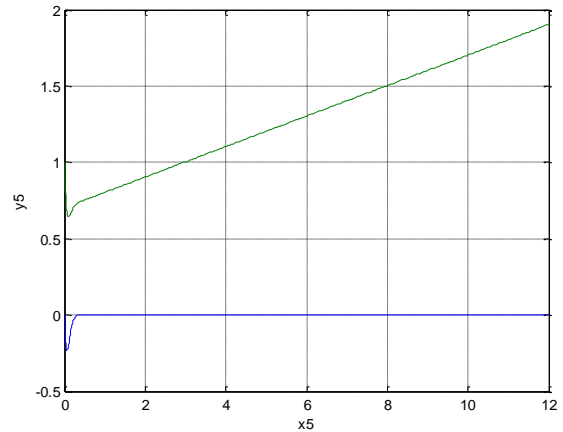


**Figure 4(c)** Synchronization between $x_5$ and $y_5$ with time t
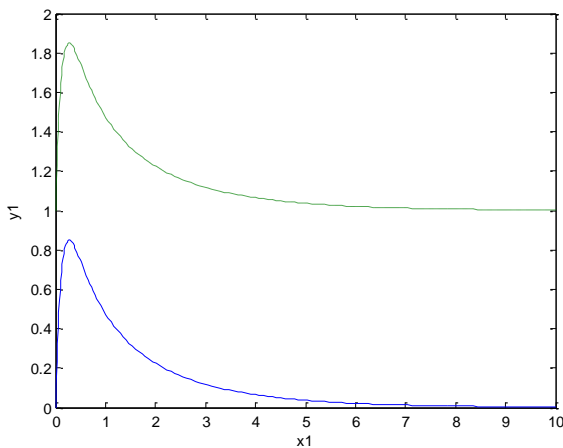


**Figure 4(a)** Synchronization between $x_1$ and $y_1$ with time t
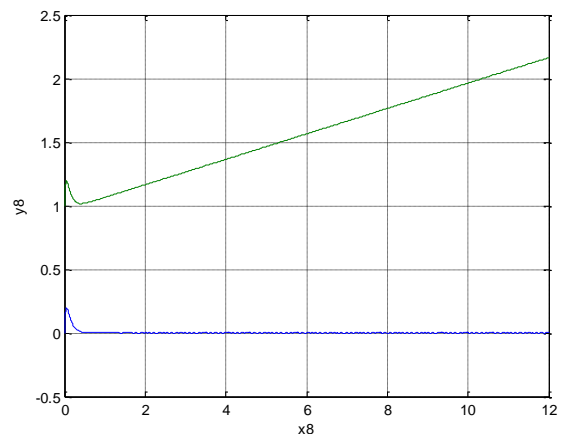


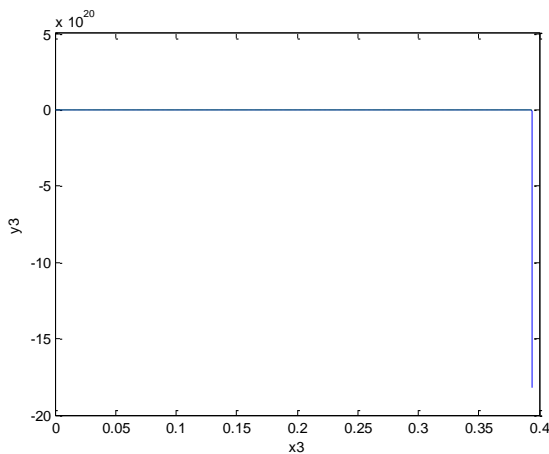**Figure 4(d)** Synchronization between $x_8$ and $y_8$ with time t



**Figure 4(b)** Synchronization between $x_3$ and $y_3$ with time t

## Conclusion

Two identical complex system are synchronized using nonlinear control method [17]. The initial conditions sensitivity and maximal Lyapunov exponent and are calculated to show the chaotic behavior of that system. Many systems as Lorenz system, Rössler system used the technique which is effective and convenient for synchronization. Numerical calculation shows good agreement to the analytical results.

## References

[1]  L. M. Pecora and T. L. Carrol, "Synchronization in chaotic systems'', Phys. Rev. Lett. 64 (8), 821, (1990).

[2]  L. M. Pecora and T. L. Carrol," Driving systems with chaotic signals", Phys. Rev. a 44, 2374, (1991).

[3] Wen Yu, "High-gain observer for chaotic synchronization and secure communication", Int. J. Commune. Syst., 18,487, (2005).

[4] Ö. Morgul and E. Solak, "Observer based synchronization of chaotic systems", Phys. Rev. E 54 (5),4803, (1996).

[5] U. E. Vincent, "Synchronization of Rikitake chaotic attractor via active control", Phys. Lett. A, 343, 133–138, 2005.

[6] U. E. Vincent, "Chaos Synchronization Using Active Control and Backstepping Control: A Comparative Analysis" Nonlinear Analysis: Modelling and Control, 253–261, 2008, Vol. 13, No. 2

[7] Parritz U., Junge L., Lauterborn W., "Experimental observation of phase synchronization", Phys. Rev. E 54 (2),2115, (1996).

[8] Tas's P. A. at al.," Detection of n:m phase locking from noisy data: Application to magneto Encephalography", Phys. Rev. Lett. 81 (15),3291, (1998).

[9] Kapitaniak T. "Controlling Chaos-Theoretical. Practical Methods in Non- linear Dynamics", Academic Pree, London.1996

[10] Sebastian Sudheer K., and Sabir M "Hybrid Synchronization of hyperchaotic Lu system", Indian Academy of Sciences, Vol. 73, No.4, 781-786, 2009.

[11] Qingming Miao, Yang Tang, Sujan Lu, Jian 'a Fang, "Synchronization Control between Two

[12] Different Chaotic Systems Synchronization Control between Two Different Chaotic Systems"

[13] Nakata S., Murata T., Ojima N., Yoshikawa K. (1998): "Self-synchronization in coupled salt-water Oscillators", Physics D, Vol. 115, pp. 313-320.

[14] Sundara Pandian V. "Hybrid Synchronization of Liu and Chen Systems", International Journal of Mathematical Sciences and Applications, Vol.1, No.3,2011.

[15] Nayak, Ganesh Chandra, R. Rajasekar, and Chapal Kumar Das. "Dispersion of SiC coated MWCNTs in PEI/silicone rubber blend and its effect on the thermal and mechanical properties." Journal of Applied Polymer Science 119.6 (2011): 3574-3581.

[16] Nayak, Ganesh Chandra, et al. "Effect of MWNTs and SiC-coated MWNTs on properties of PEEK/LCP blend." Journal of Nanotechnology 2009 (2009).

[17] Manikandan, Ayyar, et al. "Morphological, thermal, and blood-compatible properties of electrospun nanocomposites for tissue engineering application." Polymer Composites 39 (2018): E132-E139.