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Cancellation of Steady State Error Using Auxiliary Diophantine Equation for a Bio-Reactor Process -A case study

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Abstract
RST (Regulation, Sensitivity and Tracking) controller is very widely used in electrical engineering application. The controller provides both feed-forward and feedback actions. PID, Internal Model Controller (IMC) designs are mainly very effective in set point tracking but poor in disturbance rejection, however, the disturbance rejection can be obtained at the cost of reduced stability margins. The controller is based on the resolution of a Diophantine equation. A mathematical Model for Bioreactor was developed using MATLAB. Once the discrete model of the process is known and a model transfer function for the closed loop system has been chosen, the cancellation of steady state errors in response to reference signals has been solved for polynomial reference signals of any order by the introduction of an auxiliary Diophantine equation. The results are compared with the PID controller based on pole assignment method.
Keywords: *Diophantine Equation, Bio-Reactor, Steady State Error, MATLAB, PID Controller, RST Controller.*

1. Introduction
STC comprises of two coupled sub algorithms, one for the online estimation of the parameters of an assumed model and the other for evaluating the control action from a suitable control law design procedure. In principle any estimation algorithm can be combined with any control law design algorithm, thus the scope is wide and the final choice of this combination will depend on the particular application. Biofuel receives more interest lately due to the increase in oil price worldwide, and due to the green commitments that governments have taken for reducing emissions of greenhouse gases, the steam pretreatment process occurs in a pressurized continuous thermal reactor, which is preceded by a pressurization unit also known as a particle pump [1]. Depending on the load, the particle pump releases an amount of biomass to the thermal reactor with a certain frequency. An adaptive control strategy is developed based on the L1 adaptive output feedback controller. L1 adaptive control represents the latest novelty in control theory [2]. Also, a new tuning method of the L1 controller is proposed in this paper based on minimizing the integral absolute error (IAE) performance function. An on- line controller in the form of a Multi Input Multi Output Self-Tuning Regulator is designed to stabilize a 6 degree of freedom helicopter in hover state [3], [5]. A new multi - linear model approach is used to control a strongly nonlinear process in [4]. The performance of the ball and hoop system, which is difficult to control optimally using a PID controller because of the constantly changing system parameters, is presented [6].GA based optimal adaptive controller is designed for the and perturbations are applied to the system to check the robustness of the proposed system. The GASL is designed to enhance the performance of Adaptive Controllers in the Self-Tuning Regulator (STR) framework [7]. In [8], three

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self-tuning regulators for optimal control of large scale stochastic systems are developed using a generalized minimum variance approach with explicit and implicit schemes. Process modeling and transfer function is analyzed in section 2. Section 3 describes description of STR. Problem formulation and design steps are carried out in section 4&5. Section 6 shows the implementation of RST controller applied to bioreactor process. Section 7&8 overviews the design and implementation of digital PID A controller and Simulation results are discussed in section 9, followed by conclusions.

2. Process Modeling and Transfer Function

The dynamics of a completely mixed tank reactor were considered as shown in Figure 1.

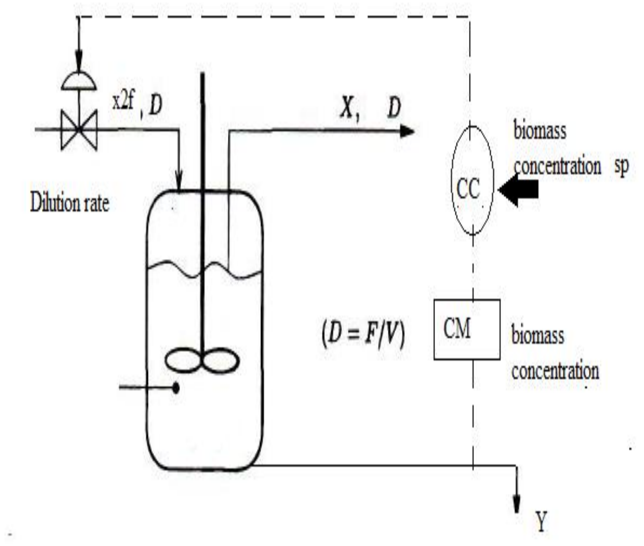

Figure 1 Schematic Representation of Bioreactor

The influent flow rate is equal to the effluent (output) flow rate i.e. F_{in}=F [volume/time]. Hence, the volume V is constant. [13-15] The influent has a substrate concentration Sin [mass/volume] and cell concentration X_{in} [mass/volume]. The rate of accumulation of biomass is obtained from a mass balance. Assuming that the biomass has a specific growth rate μ. The total amount of produced biomass per time unit in a reactor with volume V is μVX. Since the reactor is completely mixed, the outflow concentration of biomass is equal to the concentration in the tank. The rate of change of biomass is then given as

$$V\frac{dx}{dt} = \mu Vx + Fx_{in} - Fx \quad (1)$$

The dilution rate is given by equation 2

$$D = \frac{F}{V} \quad (2)$$

and the equation (1) can be written as

$$\frac{dx}{dt} = \mu x + D(x_{in} - x) \quad (3)$$

For the substrate consumption, assume that the yield coefficient is Y.

$$V\frac{dS}{dt} = F S_{in} \frac{\mu}{y} Vx - FS \quad (4)$$

Where S is substrate concentration. Equation (5) is obtained by substituting the value of D

$$\frac{dS}{dt} = -\frac{\mu}{y}x + D(S_{in} - S) \quad (5)$$

μ is the Monod function and is given by equation(6)

$$\mu = \frac{\mu_0 S}{K_m + S + K_I S^2} \quad (6)$$

The volume variation is given by equation (7)

$$\frac{dV}{dt} = F_{in} - F \quad (7)$$

A mass balance for the biomass is given by equation 8

$$\frac{d}{dt}(Vx) = \mu Vx - Fx \quad (8)$$

Dilution rate is substituted in (8) and rearranged which gives equation (9)

$$\frac{dx}{dt} = (\mu - D)x \quad (9)$$

The transfer function for a bioreactor process can be obtained by equations (5),(6) and (9).The transfer function relating Biomass concentration(x) and dilution rate (D) is obtained and is given by equation(10)

$$G_p(s) = \frac{-1.5302s - 0.459}{s^2 + 2.564s + 0.6792} \quad (10)$$

The above equation is written in inverse Z transform and is given by equation (11)

$$G_p(s) = \frac{-0.1327z^{-1} + 0.1368z^{-2}}{1 - 1.768z^{-1} + 0.7738z^{-2}} \quad (11)$$

3. Self-Tuning Regulator

Usually there are four types of adaptive control schemes: self-tuning regulators, model-reference adaptive control, gain scheduling and dual control. The block diagram of a self-tuning regulator is shown in Figure 2.

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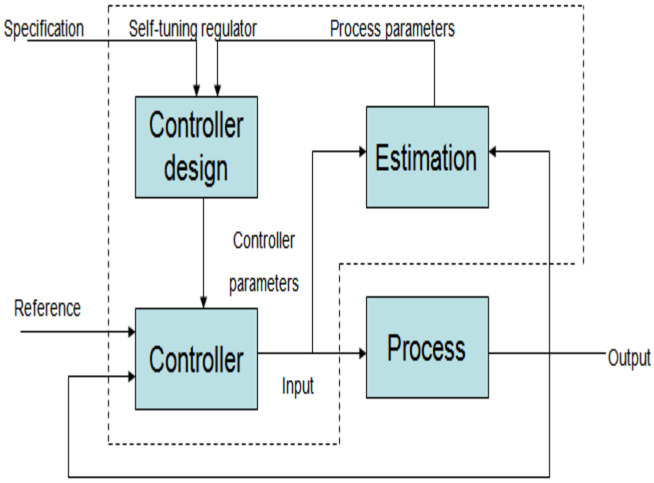

Figure 2 Schematic Representation of Self-Tuning Regulator

The "Estimator" block represents an on-line estimation of the process parameters using least-squares or projection algorithms. The block "Controller Design" represents an on-line solution to a design problem for a system with known parameters or with estimated parameters. The block "Controller" is to calculate the control action with the controller parameters computed by design. [9-12]

4. Problem Formulation

A block diagram of the closed-loop system is illustrated in the following figure 3. Let us assume that a discrete-time plant sampled at a given period Ts, with control signal u (t), measured output signal y (t) and is described in terms of z-transforms by the equation (12).

$$A(Z^{-1})y(t) = B(Z^{-1})u(t-d_0) + v(t-d_0) \qquad (12)$$

Where,

$$A(Z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots a_n z^{-n}$$

$$B(Z^{-1}) = 1 + b_1 z^{-1} + b_2 z^{-2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots b_n z^{-n}$$

With m = n- d0. In equation (12), y is the output, u is the input of the system, and v is a disturbance.

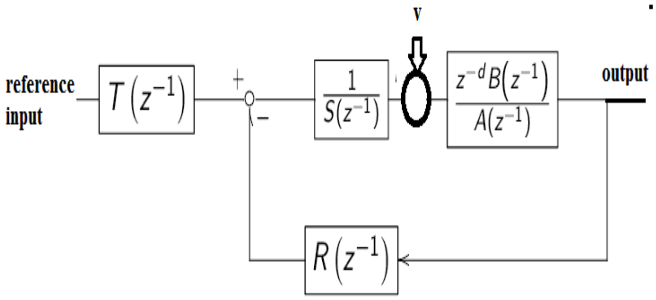

Figure 3 Adaptive Pole-Placement Control Structure Using RST Polynomials

The closed loop transfer function is given by equation (13)

$$y(t) = \frac{BT}{AR+BS} y_r(t) \qquad (13)$$

Where the problem is to compute the coefficients in the polynomials (R, S, and T) so that specified close loop is obtained.

5. Design Steps

- A Z-N method is conducted to determine the values of T_u and ωn
- The sampling time is chosen as $T_s = T_{del}$
- Based on the relay experiment the following values can be determined by using the tuning parameters of $\alpha=0.5, \beta=1, \zeta=\frac{\sqrt{3}}{2}$

$$a_0 = -exp^{-\left(\frac{2\beta T_S}{T_u}\right)} \qquad (14)$$

$$a_{m1} = -2e^{-\left(\frac{\delta\alpha 2\pi T_S}{T_u}\right) * \cos\left(\frac{\alpha 2\pi T_{s\sqrt{1-\delta^2}}}{T_u}\right)} \qquad (15)$$

$$a_{m2} = -\exp^{-\left(\frac{2\varsigma 2\pi T_S}{T_u}\right)} \qquad (16)$$

$$A_m(z) = z^2 + a_{m1}(z) + a_{m2} \qquad (17)$$

$$A_0(z) = (z + a_0)^2 \qquad (18)$$

$$A_0(z) = (z + a_0)^2$$

$$P(z) = A_0(z) A_m(z)$$

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$$T(z) = t_0 A_0(z), where\ t_0 = \frac{A_m(z=1)}{B(z=1)} \qquad (20)$$

The RST controller is defined by the following equations

$$R(z) = z^2 + r_1 z + r_2$$

$$T(z) = \frac{s_0+s_1+s_2}{1+2a_0+{a_0}^2} A_0(z) \qquad (21)$$

$$S(z) = s_0 z^2 + s_1 z + s_2$$

The process input is given by equation (22)

$$u(t) = \frac{T(q)}{R(q)} y_r(t) - \frac{S(q)}{R(q)} y(t) \qquad (22)$$

$$A(z)R(z) + B(z)S(z) = A_0(z) * A_m(z) \qquad (23)$$

Equation (23) is called as Diophantine Equation and can be written in the following matrix form and can be solved to obtain the values of R, S and T controller.

$$\begin{bmatrix} b_1 & 0 & 0 & 1 & 0 \\ b_2 & b_1 & 0 & a_1 & 1 \\ 0 & b_2 & b_1 & 0 & a_1 \\ 0 & 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} p_1 - a_1 \\ p_2 - a_2 \\ p_3 \\ p_4 \\ -1 \end{bmatrix} \qquad (24)$$

And the polynomial T (z) is chosen as

$$T(z) = t_0(z + a_{01} z + a_{02}) \qquad (25)$$

The input for the process is given by the equation

$$u(t) = \frac{1}{R}[T y_r - S y] \qquad (26)$$

Due to the integral action we have R (1) = 0

$$y = t_0 \frac{B}{A_m} y_r \qquad (27)$$

giving unity gain for

$$A(1)R(1) + B(1)S(1) = A_0(1) * A_m(1)$$

$$t_0 = \frac{s_0+s_1+s_2}{1+a_{01}+a_{02}} \qquad (28)$$

6. Implementation of RST Controller

For the identified model given by equation (12), the following values are obtained using Z-N method.

P_u=2.5; ω_n=2.51; Ku=1.44; T_s=0.1 sec.

The following values are determined by using equations (14) - (20). Equation (24) is solved to obtain the coefficients of R& S polynomials and the values of a1, a2, b1, b2, p1 p2, p3 p4 are substituted and are given by equation (29)

$$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 0.7038 & 0 & 0 & 1 & 0 \\ 0.275 & 0.7038 & 0 & -0.4979 & 1 \\ 0 & 0.275 & 0.7038 & 0 & -0.4979 \\ 0 & 0 & 0.275 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1.2109 \\ -0.1729 \\ -0.1049 \\ 0.0242 \\ -1 \end{bmatrix} \qquad (29)$$

The controller is implemented using RST polynomials and the simulated results are shown in figure 4.

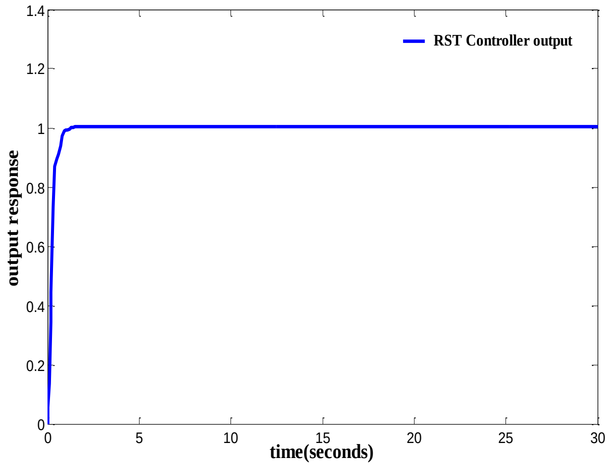

Figure 4 Representation of RST Controller Output

7. Digital PID Controllers Based on the Pole Assignment Method

A controller is designed based on the assignment of poles in a closed feedback control loop to stabilize the closed loop. A PID controller is designed which ensures the required control loop dynamic Behaviour by choosing the characteristic polynomial such that the desired transient parameters should be obtained.

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7.1. Structure of the PID–A Control Loop

General closed loop block diagram is shown in figure 5. This type of a controller was designed using the following design. The required control response of a closed loop can be achieved by selecting natural frequency ω_n and damping factor ξ in the characteristic equation for a continuous-time second-order plant

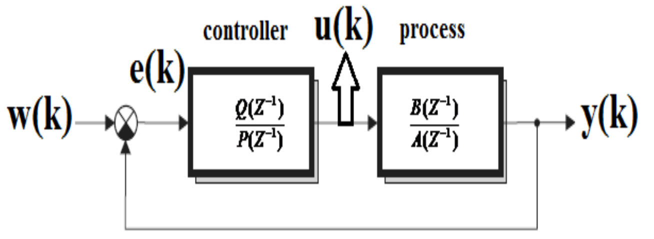

Figure 5 Block Diagram Representation of PID A Controller

A discrete transfer function of the controlled plant is given by

$$G_P(Z) = \frac{Y(Z)}{U(Z)} = \frac{B(Z^{-1})}{A(Z^{-1})} \quad (30)$$

and the polynomials are

$$A(Z^{-1}) = 1 + a_1 Z^{-1} + a_2 Z^{-2} \quad (31)$$

$$B(Z^{-1}) = b_1 Z^{-1} + b_2 Z^{-2} \quad (32)$$

The transfer function of the controller is given by

$$G_R(Z) = \frac{U(Z)}{R(Z)} = \frac{Q(Z^{-1})}{P(Z^{-1})} \quad (33)$$

Where

$$Q(Z^{-1}) = q_0 + q_1 Z^{-1} + q_2 Z^{-2} \quad (34)$$

$$P(Z^{-1}) = (1 - Z^{-1})(1 + \gamma Z^{-1}) \quad (35)$$

from equation (34), the controller equation is given by

$$U(Z) = \frac{Q(Z^{-1})}{P(Z^{-1})} E(Z) \quad (36)$$

Equation (34) & (35) are substituted in equation (36) and the controller output is obtained as

$$u(k) = q_0 + q_1 e(k-1) + q_2 e(k-2) + (1 - \gamma) u(k-1) + \gamma u(k-1)$$

Transfer function of the closed loop is given by

$$G_w(z) = \frac{Y(z^{-1})}{W(z^{-1})} = \frac{B(z^{-1})Q(z^{-1})}{A(z^{-1})P(z-1) + B(z^{-1})Q(z^{-1})}$$

(38)

Where the denominator polynomial is given by

$$D(Z^{-1}) = A(z^{-1})P(z-1) + B(z^{-1})Q(z^{-1}) \quad (39)$$

The desired pole placement is fixed for the transfer function given by the equation (30) and this can be achieved by finding the solution for the controller given by equation (33). If the polynomial D (z−1) is chosen in the form

$$D(Z^{-1}) = 1 + d_1(Z^{-1}) + d_2(Z^{-1}) \quad (40)$$

The following equations are used to calculate the coefficients for a sampling period T0

$$d_1 = -2e^{(-\varepsilon\omega_{nT_0})\cos(\omega_{nT_0\sqrt{1-\varepsilon^2}}} \text{ for } \varepsilon \le 1 \quad (41)$$

$$d_1 = -2e^{(-\varepsilon\omega_{nT_0})\cosh(\omega_{nT_0\sqrt{1-\varepsilon^2}}} \text{ for } \varepsilon > 1 \quad (42)$$

$$d_2 = e^{(-2\varepsilon\omega_n T0)} \quad (43)$$

$$x_1 = d_1 + 1 - a_1 \quad (44)$$

$$x_2 = d_2 + a_1 - a_2 \quad (45)$$

$$x_3 = a_2; x_4 = 0 \quad (46)$$

$$q_0 = \frac{1}{b_1}[d_1 + 1 - a_1 - \gamma] \quad (47)$$

$$q_1 = \frac{a_2}{b_2} - q_2\left[\frac{b_1}{b_2} - \frac{a_1}{a_2} + 1\right] \quad (48)$$

$$\gamma = q_2 \frac{b_2}{a_2} \quad (49)$$

$$\begin{bmatrix} b_1 & 0 & 0 & 1 \\ b_2 & b_1 & 0 & a_1 - 1 \\ 0 & b_2 & b_1 & a_2 - a_1 \\ 0 & 0 & b_2 & -a_2 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ \gamma \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (50)$$

8. PID a Controller Design Applied to a Bioreactor Process

PID A controller is designed for a bioreactor process using the equations (41) - (49) and is given in table 1.

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The simulated results are shown in figure 5.RST controller gave a good response compared to PID A controller.

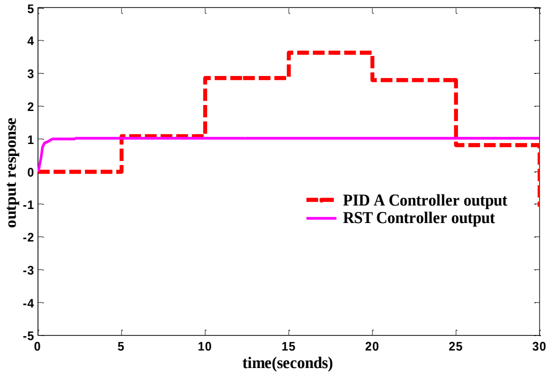

Figure 6 Comparison of PID A Controller and RST Controller Output

Table 1 Designed Values for A PID Controller for A Bioreactor Process.

S.no	Parameters	Values
1	d_1	-1.34
2	d_2	0.45
3	d_3,d_4	0
4	x_1	1.428
5	x_2	-2.0918
6	x_3	0.7738
7	x_4	0
8	q_0	-31.9
9	q_1	-10.45
10	q_2	6.96
11	γ	-2.81

9. Simulations and Discussion

This section is aimed to examine the effectiveness and performance of the proposed RST controller and PID A controller for a stable bioreactor process. After performing simulations, Figure 6 depicts the time history of the output tracking response for a step input. RST controller has better control performance over PID A controller. It can be seen that the RST controller makes the speed of response much quicker, the overshoot much smaller, and the oscillation time much shorter such that the states soon reach the level of stability than that of the PID A controller.

Conclusion

RST controller is designed for a biochemical reactor and its output response was compared with the PID A controller output. It was found that proposed controller design gave good response when compared to conventional controller and also it shows excellent reference tracking capabilities at the expense of less effort which have been applied to the field of bioreactor control. Its implementation is quite easy, and it is expected that the proposed controller will gain wide acceptance in bioprocess and fermentation control applications. Simulation results demonstrate the effectiveness of the improved control scheme.

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