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Free Vibration Analysis of Timoshenko Sandwich Beam Tapered Along Thickness and Width Resting on Pasternak Foundation

Madhusmita Pradhan[1], Nabakishor Dang[2], Pusparaj Dash[3], Prasanta Kumar Pradhan[4], Madhumita Mohanty[5]

[1] Assistant Professor, Department of Mechanical Engineering, VSSUT, Burla, Sambalpur, Odisha, India.

[2]PhD Scholar, Department of Mechanical Engineering, VSSUT, Burla, Sambalpur, Odisha, India.

[3]Professor, Department of Mechanical Engineering, VSSUT, Burla, Sambalpur, Odisha, India.

[4]Associate Professor, Department of Mechanical Engineering, VSSUT, Burla, Sambalpur, Odisha, India

[5]Faculty, School of Mechanical Engineering, KIIT DU, Bhubaneswar, India.

Email ID: *osme.madhusmita@gmail.com[1], nabakishordang@gmail.com[2], prdash_india@yahoo.co.in[3], prasant2001uce@gmail.com[4], madhumita.mohanty92@gmail.com[5]*

Abstract

This study delves into the analysis of the free vibration characteristics of a Timoshenko sandwich beam tapered along both width and thickness, subjected to an axially pulsing load on a Pasternak foundation. Integral to this investigation are the comprehensive considerations of energy expressions encompassing bending, axial deformation, kinetic energy, and strain energy induced by transverse shear stress. Subsequently, the differential equations governing the system, along with the associated boundary conditions, are deduced utilizing Hamilton's principle, followed by their non-dimensionalization. Leveraging series solutions from prior research that satisfy these equations and boundary conditions, each coordinate of the system is addressed. In the quest for a deeper understanding, Galerkin's energy principle is employed to derive matrix expressions for key parameters such as mass and stiffness. Utilizing these matrices, Eigenvalues are computed to ascertain the natural frequencies of the system. The findings are presented through a series of graphical representations, elucidating the influence of various system parameters.

Keywords: *Timoshenko Sandwich Beam; Free Vibration; Natural Frequency; Taper Parameter; Temperature Gradient.*

1. Introduction

In order to reduce noise and vibration, sandwich beam are often used in the automobile, aerospace, and marine industries. The strength to weight ratio of Timoshenko sandwich beam is higher that of solid straight beams. In order to reduce noise and vibration, it is necessary to look at the needs in numerous industries and extensive study has been done on sandwich beams. Ray and Kar [1] investigated the instability of sandwich beams across various boundary conditions, scrutinizing parametric vibrations extensively. In a separate study, Kerwin et al. [2] delved into the damping effects on the free vibration of sandwich beams, particularly focusing on cases where the core material exhibits viscoelastic behavior. Meanwhile, Saito and Otomi [3] conducted an analysis on the stability of beams supported by viscoelastic supports, incorporating the influence of mass placement along the beam's length. Additionally, Kar and Sujata [4] observed a correlation between increasing taper parameters and decreased stability against periodic forces, as well as diminished static buckling loads. The core density parameter has no impact on the static but it does have a discernible effect on the beam's sensitivity to pulsating force, either increasing or decreasing. Nayak and Dash [5] Observed that the static stability

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of the beam increases as the pre-twist angle increases, but only for odd modes. Ansari et al. [6] studied a cantilever sandwich beam with tip mass and the effect of tip mass on the stability of the system. Lin and Chen [7] employed FEM to investigate the dynamic stability of a rotating sandwich beam. Their study showed that increasing the thickness of the viscoelastic layer and angular velocity led to improved dynamic stability. However, large setting angles were found to have a negative impact on dynamic stability. Nayak et al. [8] investigated a tapered beam of exponential variation, pre-twisted system on a Pasternak foundation supported viscoelastically and a temperature grade. Pradhan et al. [9] examined the stability of an uneven sandwiched beam on a foundation of Pasternak type with a temperature grade. The stability of a non-uniform uneven sandwich beam on a Pasternak base in a temperature environment was investigated by Pradhan et al. [10]. Malekzadeh et al [11] analysed on a functionally graded circular curved beam and found that the frequency parameter decreases as the opening angle increases. The dynamic Timoshenko beam subjected to axial periodic force was studied by Sabuncu and Evran [12]. The researchers concluded that as the distance between the centroid and the centre of flexure rises, the coupled bending dominant frequency grows. Ditaranto and Blasingame [13] found generalized result for composite loss factor and natural frequencies. The forced vibration was analysed by Mead and Marcus [14] for different boundary conditions by using the solutions provided by Ditaranto and observed different uncoupled complex modes. Banerjee [15] presented the problem-solving approach by dynamic stiffness method for sandwich beam. Nayak et al. [16] investigated a tapered beam of exponential variation, pre-twisted system on a Pasternak foundation supported viscoelastically and a temperature grade. A functionally graded, pre-twisted thick cantilever beam was the subject of an investigation by Mohanty et al. [17] into its vibration and dynamic stability. They discovered that a low index damages the system's stability when power levels rise. Liao & Huang [19] enriched the literature by exploring the parametric instability of spinning pre-twisted beams under periodic axial forces, contributing to a deeper comprehension of the intricate phenomena dictating these structures. In a complementary study, Shiau & Tong [20] explored the intricate relationship between stability and response in rotating pre-twisted tapered blades, shedding light on the nuanced dynamics at play. Joubaneh et al. [21] conducted numerical and experimental analyses of free vibration in sandwich beams with tip masses using higher-order theory, highlighting the significant impact of tip masses on natural frequency. Freidani et al. [22] considered the consequence of a moving mass and curvature radius on dynamic deflection, observing substantial increases with higher parameter values. As investigation of free vibration analysis of an asymmetric Timoshenko sandwich beam which has tapered along depth subjected to axial pulsating excitation is not available till date, the existing work shares out with the above said system configuration

2. Problem Formulation

Figure 1 shows a 3-layer Timoshenko tapered sandwiched beam with a periodic axial load. The periodic axial load is denoted by $P(t) = P_0 + P_1 \cos \omega t$.

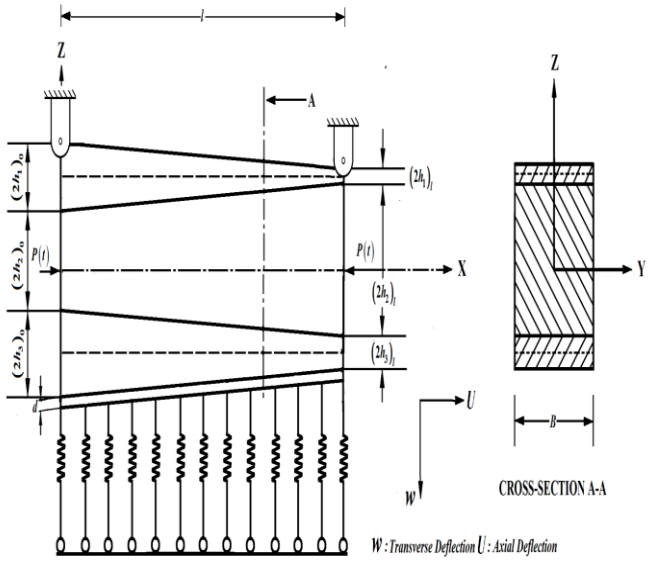

Figure 1 System Configuration

2.1 Materials

In this analysis, a three-layer sandwich beam is constructed using plywood as the viscoelastic middle layer, with structural steel and Cu-Al alloy being considered for the top and bottom sheets,

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respectively. Using the right adhesive, the layers are joined. The materials used to build the beam are listed in Table 1 along with a variety of mechanical attributes.

Table 1 Mechanical characteristics of the Considered Materials

Materials	Density (kg/m³)	highest tensile strength (MPa)	Young's modulus (MPa)
ASTM A36 Steel, plate	7800	550	200*10³
Cu-Al Alloy CuAl8Fe3	7850	470	170*10³
Materials	**Density (kg/m³)**	**the highest tensile strength (Pa)**	**Young's modulus (Pa)**
Plywood	650	13.8	620*10³

The equations of motion for the beam are formulated under specific assumptions, which are as follows.

1. The elastic layers of the skin adhere to Euler's theory, while the beam's core undergoes shear deformation theory
2. The system's pre-twist angle is incredibly modest.
3. It is presumed that there is no sliding at the intersection of the beam's several levels.
4. It is possible to disregard the strain energy and extensional inertia of the core caused by in-plane stresses.
5. The beam faces' rotational inertia may be minimal.
6. The deflection curves have short slopes. This is owing to the expectation that any deflection will be constrained by an increase in the stiffness of the system brought on by a high elastic modulus or a thicker viscoelastic layer.
7. It is deemed possible to ignore the resulting force in the intermediate viscoelastic layer of the beam. Its elastic modulus is far lower than those elastic layers.

The total kinetic energy of the system can be written by

$$T=\frac{1}{2}\int_0^l (M\dot{w}^2+\rho_1A_1\dot{u}_1^2+\rho_3A_3\dot{u}_3^2+\rho_1I_1\dot{\theta}_1^2+\rho_3I_3\dot{\theta}_3^2)dx \quad (1)$$

The total potential energy of the system can be written by

$$V=\frac{1}{2}\int_0^l\left[\begin{pmatrix}E_1A_1u_{1,x}^2+E_3A_3u_{3,x}^2+E_1I_1\theta_{1,x}^2+E_3I_3\theta_{3,x}^2\\ +K_1A_1G_1\left(w_{,x}-\theta_1\right)^2+K_2A_2G_2\gamma_2{}^2+K_3A_3G_3\left(w_{,x}-\theta_3\right)^2\end{pmatrix}\right]dx+$$

$$\frac{1}{2}G_sbd\int_0^l w_{,x}^2dx+\frac{B}{2}\int_0^l k(x)w^2dx \quad (2)$$

The total work done of the system can be written by

$$W=\frac{1}{2}\int_0^l P(t)w_{,x}{}^2dx \quad (3)$$

Energy principle by Hamilton $\delta\int_{t_1}^{t_2}(T-V-W)dt=0$, the following are the governing

$$-M\ddot{w}+K_1A_1G_1w_{,xx}+K_1A_1G_1\theta_{1,x}+K_3A_3G_3w_{,xx}+K_3A_3G_3\theta_{3,x}+G_sbdw_{,xx}-Bk(x)w-P(t)w_{,xx}=0 \quad (4)$$

$$-(\rho_1A_1+\alpha^2\rho_3A_3)\ddot{u}_1+(E_1A_1+\alpha^2E_3A_3)u_{1,xx}-\frac{K_2A_2G_2}{4h_2{}^2}\left[(1+\alpha^2)u_1-(1+\alpha)h_1\theta_1-(1+\alpha)h_3\theta_3\right]=0 \quad (5)$$

$$(-\rho_1I_1\ddot{\theta}_1+E_1I_1\theta_{1,xx}+K_1A_1G_1w_{,x}-K_1A_1G_1\theta_1-\frac{K_2A_2G_2}{4h_2{}^2}\left[h_1{}^2\theta_1-(1+\alpha)h_1u_1+h_1h_3\theta_3\right]=0 \quad (6)$$

$$(-\rho_3I_3\ddot{\theta}_3+E_3I_3\theta_{3,xx}+K_3A_3G_3w_{,x}-K_3A_3G_3\theta_3-\frac{K_2A_2G_2}{4h_2{}^2}\left[h_3{}^2\theta_3-(1+\alpha)h_3u_1+h_1h_3\theta_1\right]=0 \quad (7)$$

and the boundary conditions for the system are

$$-K_1A_1G_1w_{,x}+K_1A_1G_1\theta_1-K_3A_3G_3w_{,x}+K_3A_3G_3\theta_3-G_sbdw_{,x}+P(t)w_{,x}=0 \quad (8)$$

$$E_1A_1u_{1,x}+\alpha^2E_3A_3u_{1,x}=0 \quad (9)$$

$$E_1I_1\theta_{1,x}=0 \quad (10)$$

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$$E_3 I_3 \theta_{3,x} = 0 \tag{11}$$

The dimensionless form of total energy terms are as follows

$$T = \frac{1}{2}\int_0^1 \bar{M}\dot{\bar{w}}^2 d\bar{x} + \frac{h_{10}B\rho_{10}}{M_{10}}\int_0^1 \bar{\rho}_1 \bar{A}_{1x}\dot{\bar{u}}_1^{\,2} d\bar{x} + \frac{h_{10}B\rho_{10}\alpha^2}{M_{10}}\int_0^1 \bar{\rho}_3 \bar{A}_{3x}\dot{\bar{u}}_1^{\,2} d\bar{x} + \frac{Bh_{10}^{\;3}\rho_{10}\theta_0^{\,2}}{3M_{10}L^2}\int_0^1 \bar{\rho}_1 \bar{I}_1 \dot{\bar{\theta}}_1^{\,2} d\bar{x} + \frac{Bh_{10}^{\;3}\rho_{10}\theta_0^{\,2}}{3M_{10}L^2}\int_0^1 \bar{\rho}_3 \bar{I}_3 \dot{\bar{\theta}}_3^{\,2} d\bar{x} \tag{12}$$

$$\begin{aligned}
V = {} & \frac{3L^2}{2h_{10}^{\;2}}\int_0^1 \bar{E}_1 \bar{A}_1 \bar{u}_{1,\bar{x}}^{\;2} d\bar{x} + \frac{3L^2\alpha^2}{2h_{10}^{\;2}}\int_0^1 \bar{E}_3 \bar{A}_3 \bar{u}_{1,\bar{x}}^{\;2} d\bar{x} + \frac{\theta_0^{\,2}}{2}\int_0^1 \bar{E}_1 \bar{I}_1 \bar{\theta}_{1,\bar{x}}^{\;2} d\bar{x} \\
& + \frac{\theta_0^{\,2}}{2}\int_0^1 \bar{E}_3 \bar{I}_3 \bar{\theta}_{3,\bar{x}}^{\;2} d\bar{x} + \frac{3L^2\theta_0^{\,2}}{2h_{10}^{\;2}}\int_0^1 K_1 \bar{A}_1 \bar{G}_1 (\bar{w}_{,x} - \bar{\theta}_1)^2 d\bar{x} \\
& + \frac{3L^2}{8\bar{h}_2^{\,2} h_{10}^2}\int_0^1 K_2 \bar{A}_2 \bar{G}_1 \begin{bmatrix} \left(1+\alpha^2+2\alpha\right)\bar{u}_1^2 + h_1^2\bar{\theta}_1^{\,2}\theta_0^2 + h_3^2\bar{\theta}_3^{\,2}\theta_0^2 + 2\bar{h}_1\bar{h}_3\bar{\theta}_1\bar{\theta}_3\theta_0^2 \\ -2\bar{h}_1\bar{u}_1\bar{\theta}_1\theta_0 - 2\bar{h}_1\bar{u}_1\alpha\bar{\theta}_1\theta_0 - 2\bar{h}_3\bar{u}_1\bar{\theta}_3\theta_0 - 2\alpha\bar{h}_3\bar{u}_1\bar{\theta}_3\theta_0 \end{bmatrix} d\bar{x} \\
& + \frac{3L^2\theta_0^{\,2}}{2h_{10}^{\;2}}\int_0^1 K_3 \bar{A}_3 \bar{G}_3 (\bar{w}_{,x} - \bar{\theta}_3)^2 d\bar{x} + \\
& \frac{3\bar{b}\bar{d}\bar{G}_s L^4}{4Bh_{10}^{\;3}}\int_0^1 \bar{w}_{,\bar{x}}^{\;2} d\bar{x} + \frac{3k_{10}L^4}{4E_{10}h_{10}^{\;3}}\int_0^1 \bar{k}(x)\bar{w}^2 d\bar{x}
\end{aligned} \tag{13}$$

The total dimensionless work done becomes

$$W = \frac{1}{2}\int_0^1 \bar{P}(t)\bar{w}_{,\bar{x}}^{\;2} d\bar{x} \tag{14}$$

The dimensionless governing equations for motion are

$$-\bar{M}\ddot{\bar{w}} + K_1\bar{A}_1\bar{G}_1 w_{,xx} + K_1\bar{A}_1\bar{G}_1\bar{\theta}_{1,x} + K_3\bar{A}_3\bar{G}_3\bar{w}_{,xx} + K_3\bar{A}_3\bar{G}_3\bar{\theta}_{3,x} + \bar{G}_s\bar{b}\bar{d}\bar{w}_{,xx} - \bar{B}k(\bar{x})\bar{w} - \bar{P}(\bar{t})\bar{w}_{,xx} = 0 \tag{15}$$

$$-(\bar{\rho}_1\bar{A}_1 + \alpha^2\bar{\rho}_3\bar{A}_3)\ddot{\bar{u}}_1 + (\bar{E}_1\bar{A}_1 + \alpha^2\bar{E}_3\bar{A}_3)\bar{u}_{1,xx} - \frac{K_2\bar{A}_2\bar{G}_2}{4\bar{h}_2^{\,2}}\left[(1+\alpha^2)\bar{u}_1 - (1+\alpha)\bar{h}_1\bar{\theta}_1 - (1+\alpha)\bar{h}_3\bar{\theta}_3\right] = 0 \tag{16}$$

$$(-\bar{\rho}_1\bar{I}_1\ddot{\bar{\theta}}_1 + \bar{E}_1\bar{I}_1\bar{\theta}_{1,xx} + K_1\bar{A}_1\bar{G}_1\bar{w}_{,x} - K_1\bar{A}_1\bar{G}_1\bar{\theta}_1 - \frac{K_2\bar{A}_2\bar{G}_2}{4\bar{h}_2^{\,2}}\left[\bar{h}_1^{\,2}\bar{\theta}_1 - (1+\alpha)\bar{h}_1\bar{u}_1 + \bar{h}_1\bar{h}_3\bar{\theta}_3\right] = 0 \tag{17}$$

$$(-\bar{\rho}_3\bar{I}_3\ddot{\bar{\theta}}_3 + \bar{E}_3\bar{I}_3\bar{\theta}_{3,xx} + K_3\bar{A}_3\bar{G}_3\bar{w}_{,x} - K_3\bar{A}_3\bar{G}_3\bar{\theta}_3 - \frac{K_2\bar{A}_2\bar{G}_2}{4\bar{h}_2^{\,2}}\left[\bar{h}_3^{\,2}\bar{\theta}_3 - (1+\alpha)\bar{h}_3\bar{u}_1 + \bar{h}_1\bar{h}_3\bar{\theta}_1\right] = 0 \tag{18}$$

2.2 Solution by Variation Method

The system's solution was estimated through the use of the following set of equations

$$\bar{w}(x,t) = \sum_{i=1}^{e} q_i w_i \tag{19}$$

$$\bar{u}_1(x,t) = \sum_{k=e+1}^{f} q_k u_{1k} \tag{20}$$

$$\bar{\theta}_1(x,t) = \sum_{m=f+1}^{g} q_m \theta_{1m} \tag{21}$$

$$\bar{\theta}_3(x,t) = \sum_{o=g+1}^{h} q_o \theta_{3o} \tag{22}$$

The generalised coordinates are w_i, u_{1k}, θ_{1m}, θ_{3o}

The equations (19) - (22) provide the expressions for the shape functions that correspond to the pinned-pinned boundary condition as in [12].

$$w_i(\bar{x}) = \sin(i\pi\bar{x}) \tag{23}$$

$$\bar{u}_1 = \cos(k\pi\bar{x}) \tag{24}$$

$$\bar{\theta}_1 = \sin(m\pi\bar{x}) \tag{25}$$

$$\bar{\theta}_3 = \sin(o\pi\bar{x}) \tag{26}$$

After non-dimensionalising, the extended Galerkin method is applied to the energy equations to obtain the following matrix equation

$$[M_g]\{\ddot{Q}\} + [[K] - \bar{F}_0[H]]\{Q\} - \bar{F}_1\cos\bar{\omega}\bar{t}[H]\{Q\} = \{\varphi\} \tag{27}$$

M_g and K_g are global mass matrix and global stiffness matrix correspondingly.

$$\bar{F}_0 = \frac{F_0}{f_0} \tag{28}$$

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$$\bar{F}_1(t) = \frac{F_1 \cos(\omega_f t)}{f_0} \quad (29)$$

The various sub matrices are

$$m_{11ij} = \int_0^1 \bar{M} w_i w_j d\bar{x} \quad (30)$$

$$m_{22kl} = \int_0^1 \lambda_1(\bar{\rho}_1 \bar{A}_1 + \alpha^2 \bar{\rho}_3 \bar{A}_3) u_{1k} u_{1l} d\bar{x} \quad (31)$$

$$m_{33mn} = \int_0^1 \lambda_2 \bar{\rho}_1 \bar{I}_1 \theta_{1m} \theta_{1n} d\bar{x} \quad (32)$$

$$m_{44op} = \int_0^1 \lambda_2 \bar{\rho}_3 \bar{I}_3 \theta_{3o} \theta_{3p} d\bar{x} \quad (33)$$

$$k_{11ij} = \int_0^1 [(\lambda_5 K_1 \bar{A}_{1x} \bar{G}_1 w_i' w_j') + (\lambda_5 K_3 \bar{A}_{3x} \bar{G}_3 w_i' w_j') + (\lambda_7 w_i' w_j') + (\lambda_8 \bar{k}(x) w_i w_j)] d\bar{x} \quad (34)$$

$$k_{13mj} = \int_0^1 -\lambda_5 K_1 \bar{A}_{1x} \bar{G}_1 w_i' \theta_{1m} d\bar{x} \quad (35)$$

$$k_{14oj} = \int_0^1 -\lambda_5 K_3 \bar{A}_{3x} \bar{G}_3 w_i' \theta_{3o} d\bar{x} \quad (36)$$

$$k_{22kl} = \int_0^1 [\{\lambda_3(\bar{E}_1 \bar{A}_{1x} + \alpha^2 \bar{E}_{3x} \bar{A}_{3x}) u_{1k}' u_{1l}'\} + \{\lambda_6 K_2 \bar{A}_{2x} \bar{G}_2 (1+\alpha)^2 u_{1k} u_{1l}\}] d\bar{x} \quad (37)$$

$$k_{23ml} = \int_0^1 -(1+\alpha)\lambda_6 K_2 \bar{A}_{2x} \bar{G}_2 \bar{h}_1 \theta_0 u_{1k} \theta_{1m} d\bar{x} \quad (38)$$

$$k_{24ol} = \int_0^1 -(1+\alpha)\lambda_6 K_2 \bar{A}_{2x} \bar{G}_2 \bar{h}_3 \theta_0 u_{1k} \theta_{3o} d\bar{x} \quad (39)$$

$$k_{33mn} = \int_0^1 [(\lambda_4 \bar{E}_1 \bar{I}_1 \theta_{1m}' \theta_{1n}') + (\lambda_5 K_1 \bar{A}_{1x} \bar{G}_1 \theta_{1m} \theta_{1n}) + (\lambda_6 K_2 \bar{A}_{2x} \bar{G}_2 \bar{h}_1^2 \theta_0^2 \theta_{1m} \theta_{1n})] d\bar{x} \quad (40)$$

$$k_{34on} = \int_0^1 \lambda_6 K_2 \bar{A}_{2x} \bar{G}_2 \bar{h}_1 \bar{h}_3 \theta_0^2 \theta_{1m} \theta_{3o} d\bar{x} \quad (41)$$

$$k_{44op} = \int_0^1 [(\lambda_4 \bar{E}_3 \bar{I}_3 \theta_{3o}' \theta_{3p}') + (\lambda_5 K_3 \bar{A}_{3x} \bar{G}_3 \theta_{3o} \theta_{3p}) + (\lambda_6 K_2 \bar{A}_{2x} \bar{G}_2 \bar{h}_3^2 \theta_0^2 \theta_{3o} \theta_{3p})] d\bar{x} \quad (42)$$

$$H_{11ij} = \int_0^1 w_i' w_j' d\bar{x} \quad (43)$$

2.3 Free Vibration Analysis

For determination of the natural frequencies $P(t)$ is occupied as zero (i.e., $F_0 = F_1 = 0$) in (27). The natural frequencies can be obtained from the Eigen values of the $m^{-1}k$ matrix. Referring [13] from above the following equations can be derived

3. Result and Discussion

Numerical data are available for parameters such as shear, thermal gradient, taper, spring characteristics, and spring loss factors. Figures 2 to 8 depict the influence of various system parameters on the structure's free vibration.

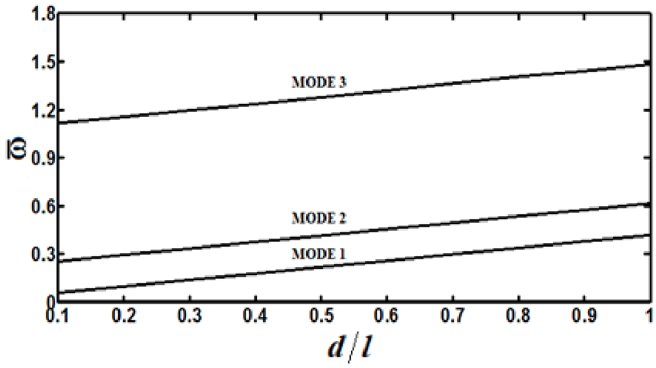

Figure 2 Response of $\bar{\omega}$ for d/l

Figure 3 depicts the impact of the d/l on the natural frequencies of the system. For larger vales of d/l, the natural frequencies of the system increase.

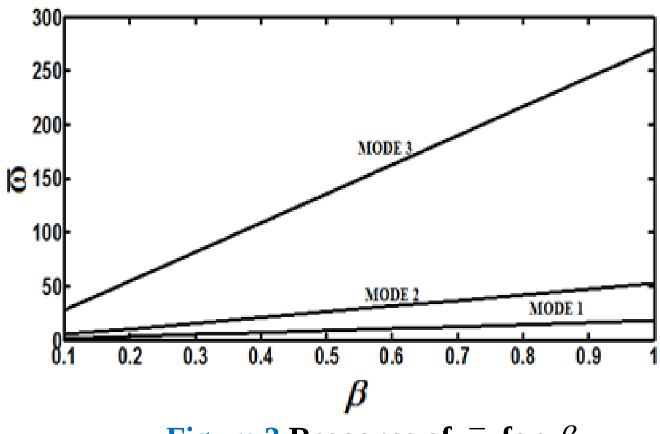

Figure 3 Response of $\bar{\omega}$ for β

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Figure 3 depicts the impact of the width tapper β on the natural frequencies of the system. For larger vales of β , the natural frequencies of the system increase.

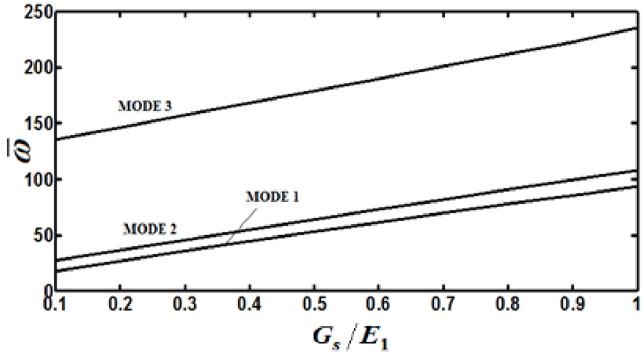

Figure 4 **Response of $\bar{\varpi}$ for G_S/E_1**

Figure 4 The G_S/E_1 outcome enhances the natural frequencies across all the three modes due to the improved rigidity of the system.

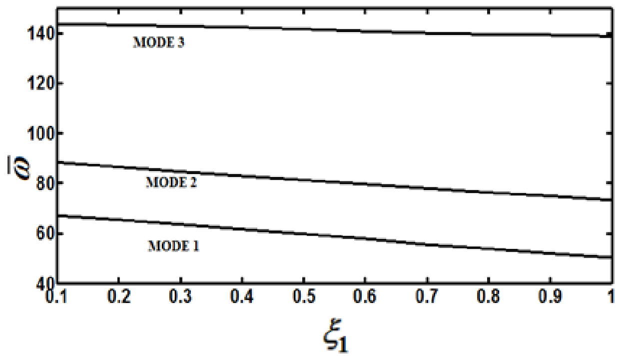

Figure 5 **Response of $\bar{\varpi}$ for ξ_1**

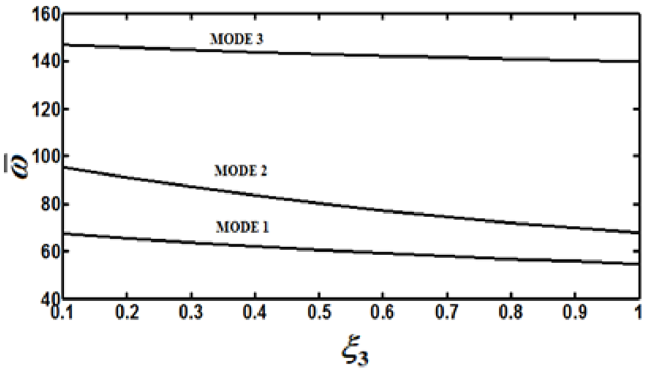

Figure 6 **Response of $\bar{\varpi}$ for ξ_3**

Figures 5 and 6 illustrate the change in natural frequencies with increasing values of the thickness taper parameter. It's evident that as the taper parameter rises, the natural frequencies decrease. This decline is attributed to the reduction in cross-sectional area, which leads to a slenderizing effect on the beam and consequent deterioration in natural frequencies.

Conclusion

The findings derived from the comprehensive study shed light on the positive correlation between the width taper and the enhancement of the beam's natural frequency. Additionally, it was observed that the ratios involving the thickness of the shear layer to the length of the beam and the modulus of the shear layer to the modulus of elasticity of the elastic layer play pivotal roles in augmenting the natural frequencies. On the contrary, elevating the taper parameter along the thickness was found to have a detrimental effect on the natural frequency, while a corresponding increase along the width was associated with improvements in the system's natural frequencies.

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