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Free Vibration Analysis of a Rotating, Dimensionally Varying Sandwich Beam Subjected to A Pulsating Axial Load

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Abstract

This study aims to utilize computational techniques to analyse the natural vibration characteristics of a rotating asymmetric sandwich beam with varying width and thickness, subjected to a pulsating axial load. The equation of motion and boundary conditions are derived employing the extended Hamilton's concept and subsequently non-dimensional zed using the extended Galerkin method. Hills equations are then obtained, and Galerkin's energy principle is applied to formulate matrix expressions for crucial parameters like mass and stiffness. These matrices are used to compute Eigenvalues, determining the natural frequencies. The results are presented graphically to illustrate the impact of different system parameters, such as taper parameters, shear parameter, rotational speed, and core-loss factor. MATLAB is utilized for graphical representation and analysis.

Keywords: *Free Vibration; Natural Frequency; Sandwich Beam; Taper Parameter.*

1. Introduction

The study of sandwich beams' vibration study has gained significant importance, especially in applications where high strength-to-weight ratios are crucial, such as space vehicles, airplanes, military aircraft, and ships. These beams, often used as load-bearing elements, find wide applications in various mechanical fields. Understanding their vibration characteristics, including natural frequencies and mode shapes, is essential for effective structural design. Sandwich beams typically consist of three layers: upper and bottom face materials and a core material. The faces endure both compressive and tensile loads and provide bending strength to the structure, while the core supports the faces and prevents deformation. Viscoelastic cores are preferred for their effective support and damping properties compared to other core materials. The choice of materials for sandwich beam construction depends on desired properties such as strength, temperature resistance, and surface finish, tailored to specific applications. A well-designed sandwich beam ensures structural integrity throughout its lifespan, effectively bearing loads. Viscoelastic materials exhibit a combination of elastic and viscous properties. They return to their original shape after stress is released (elastic behavior), but they also exhibit flow under sustained stress (viscous behavior). Creep, observed in viscoelastic materials under constant strain, and hysteresis, occurring due to cyclic loading, contribute to their damping properties. Faraday [1] gave a very important remark from his experiment on the parametric excitation, while performing the experiment he used a container filled with fluid when Faraday gave vibration vertically on the container the inside fluid surface oscillates at half of the frequency of the container. Heteny [2] went into greater depth about the concept of beams on elastic foundations. Madhusmita Pradhan [3]

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considered an asymmetric sandwich beam tapered which placed under a Pasternak foundation and pulsating load applied on it after that by using different parameters both the static and the dynamic stability measured. Yokoyama [4]as the FEM(finite element method) a today's most popular method to calculate the boundary condition, the Timoshenko beam was place in the elastic base and the both dynamic and static boundary condition examine and the effect of the base also shown on this Dinget al. [5] Many no of research carried out on the field of free vibration by many researchers which are listed below. [6]The frequency response curve of a non-uniform beam experiencing nonlinear oscillation is estimated mathematically using the various timeline approach, yielding approximate but accurate data. [7] Motohirosato explained when a beam having equal distance elastic support it will take as a elastic foundation and my the computational method the stability also calculated. [8] Hu Dinga shows how the Galerkin technique for calculating the It is possible to improve the dynamic behavior of an elastic beam sitting on a non-linear base using viscous damping.[9] E.Babilo investigates beam which is basically supported and undergo to axial are liant on time the dynamic stability calculated with the time. [10-12] Functionally graded material is defined as a continuous variation of material qualities.[13]Saito and Otomi mentioned lots of conditions as well as cases to plot the stability diagram for both static and the dynamic stability analysis which is a key point to our project point of view. By taking the taper parameters of sandwich beam and the taper width of the sandwich beam was examine with respect to the loading in both the static and dynamic condition for the different boundary condition. From these the related research papers were listed below [14] Hetenyi was the first to explain the notion of beams on elastic foundations in detail. Kerwin [15] offered a basic analysis of damping by a visco-elastic layer that is damping restricted. [16] Wang and Stephens examined the variation of natural frequencies in a Timoshenko beam with rotating inertia, shear deformation, and elastic foundation constants. Clementi et al. [17] Using the multiple time scale technique analytically, the frequency response curves of a non-uniform beam exhibiting nonlinear oscillations were derived. Caruntu [18-20]. The basic resonance of non-uniform non-uniform beams had rectangular cross section, uniform width, and convex parabolic thickness variation was investigated using forced and undammed bending vibrations (Figure 1).

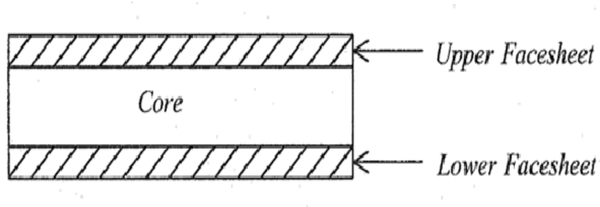

Figure 1 Sandwich Beam Layers

2. Problem Formulation

Figure 2 shows a 3-layer Non uniform sandwich beam tapered along width and thickness with a periodic axial load. The periodic axial load is denoted by $P(t) = P_0 + P_1 \cos \omega t$.

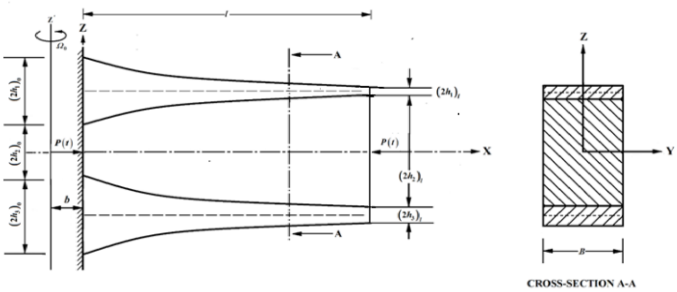

Figure 2 System Configuration

2.1 Materials

In this study, a sandwich beam comprising three layers is created, featuring plywood as the viscoelastic core, and structural steel and Cu-Al alloy for the upper and lower sheets, respectively. Employing suitable adhesive, the layers are bonded together. The following are the expressions for potential energy, kinetic energy, and work done;

$$V = \frac{1}{2}\int_0^l E_{1,x}A_{1,x}U_{1,x}^2 dx + \frac{1}{2}\int_0^l E_{3,x}A_{3,x}U_{3,x}^2 dx + \frac{1}{2}\int_0^l \left(E_{1,x}I_{1,x} + E_{3,x}I_{3,x}\right) w_{,xx}^2 dx + \frac{1}{2}G_2^*\int_0^l A_{2,x}\gamma_2^2 dx \quad (01)$$

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$$T=\frac{1}{2}\int_0^l \bar{m}w,_t^2dx+\frac{1}{2}\Omega_0^2\int_0^l\left[\bar{m}(b+x)\int_0^x w,_x^2\,dx\right]dx+\frac{1}{2}\int_0^l \bar{m}\Omega_0^2w^2dx \quad (02)$$

$$W_p=\frac{1}{2}\int_0^l p(t)w,_x{}^2\,dx \quad (03)$$

By putting Hamilton's idea into practice,

$$\delta\delta\int_{t_1}^{t_2}(T-V+W_p)\,dt=0 \quad (04)$$

To determine equations of motion, the following assumptions are used.

1. The beam has transverse deflection is modest also during the course of a certain cross section it is uniform.
2. The layers are properly linked so that there is no slippage between them.
3. The metallic layer follows the Euler-Bernoulli beam theory assumption.
4. The core's extensional and bending effects are insignificant.
5. Shear is the primary cause of core damping.
6. The effects of rotary inertia are minimal inside the layer.
7. There is no axial inertia in the higher and lower layers.

By using the extended Hamilton's principle along with the generalized Galerkin's method, the following non-dimensional equations of motion are obtained.

$$\bar{m}\bar{w},_{\bar{t}\bar{t}}+\left[1+\frac{\lambda_0^2(1+E_{31}h_{31}^3)}{(lh_{10})^2(1+E_{31}h_{31})}\left\{\frac{f}{l^2}-(\bar{x}+\bar{b})^2\right\}\right]\bar{w},_{\bar{x}\bar{x}\bar{x}\bar{x}}-\frac{2\lambda_0^2(1+E_{31}h_{31}^3)}{(lh_{10})^2(1+E_{31}h_{31})}(\bar{x}+\bar{b})\bar{w},_{\bar{x}\bar{x}\bar{x}}$$
$$+\left[-\frac{\lambda_0^2(1+E_{31}h_{31}^3)}{(lh_{10})^2(1+E_{31}h_{31})}-\lambda_0^2\left\{\frac{f}{l^2}-(\bar{x}+\bar{b})^2\right\}-3g^*\left(1+\frac{h_{12}+h_{32}}{2}\right)^2+\bar{p}(\bar{t})\right]\bar{w},_{\bar{x}\bar{x}} \quad (05)$$
$$+\lambda_0^2(\bar{x}+\bar{b})\bar{w},_{\bar{x}}+\frac{3}{2}g^*lh_{10}h_{12}\left(1+\frac{h_{12}+h_{32}}{2}\right)(1+\alpha)\frac{2(h_2)_0}{C}\gamma_{2,\bar{x}}=0$$

$$\frac{2(h_2)_0}{C}\gamma_{2,\bar{x}\bar{x}}-\frac{g^*}{4}h_{12}^2\left(\frac{1+E_{31}h_{31}^3}{1+\alpha^2E_{31}h_{31}}\right)(1+\alpha)\left[(1+\alpha)\frac{2(h_2)_0}{C}\gamma_2-\left(\frac{2(1+((h_{12}+h_{32})/2))}{(lh_{10}h_{12})}\right)\bar{w},_{\bar{x}}\right]=0 \quad (06)$$

Boundary conditions for the sandwich beam as follow At and, $\bar{x}=0,\bar{x}=1$

$$\left[1+\frac{\lambda_0^2(1+E_{31}h_{31}^3)}{(lh_{10})^2(1+E_{31}h_{31})}\left\{\frac{f}{l^2}-(\bar{x}+\bar{b})^2\right\}\right]\bar{w},_{\bar{x}\bar{x}\bar{x}}-\frac{2\lambda_0^2(1+E_{31}h_{31}^3)}{(lh_{10})^2(1+E_{31}h_{31})}(\bar{x}+\bar{b})\bar{w},_{\bar{x}\bar{x}}=0 \quad (07)$$

$$\bar{w},_{\bar{x}}=0 \quad (08)$$

$$\left[\frac{\lambda_0^2(1+E_{31}h_{31}^3)}{(lh_{10})^2(1+E_{31}h_{31})}-\lambda_0^2\left\{\frac{f}{l^2}-(\bar{x}+\bar{b})^2\right\}-3g^*\left(1+\frac{h_{12}+h_{32}}{2}\right)^2+\bar{p}(\bar{t})\right]\bar{w},_{\bar{x}}=0 \quad (09)$$

$$\bar{w}=0 \quad (10)$$

$$\frac{3}{2}g^*lh_{10}h_{12}\left(1+\frac{h_{12}+h_{32}}{2}\right)(1+\alpha)\frac{2(h_2)_0}{C}\gamma_2,_{\bar{x}}=0 \quad (11)$$

$$\gamma_2=0 \quad (12)$$

2.2 Approximate Solution

Equations (05) and (06) are assumed to have solutions

$$\bar{w}(\bar{x},\bar{t})=\sum_{i=1}^{i=p}w_i(\bar{x})f_i(\bar{t}) \quad (13)$$

$$\bar{\gamma}_2(\bar{x},\bar{t})=\sum_{k=p+1}^{k=2p}\gamma_{2k}(\bar{x}) \quad (14)$$

Here the shape functions are w_i and γ_{2k} and the generalized coordinates are f_i and f_k. The following matrix equations of motion in generalized

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coordinates are obtained by substituting the aforementioned equations in equations (05) and (06) and using the general Galerkin's method.

$$[m]\{\ddot{Q}_1\}+[k_{11}]\{Q_1\}+[k_{12}]\{Q_2\}=\{0\} \quad (15)$$

$$[k_{21}]\{Q_1\}+[k_{22}]\{Q_2\}=\{0\} \quad (16)$$

Where

$$\{Q_1\}=\{f_1,..............., f_p\}^T \quad (17)$$

$$\{Q_2\}=\{f_{p+1},..............., f_{2p}\}^T \quad (18)$$

The sub matrixes are listed below

$$m_{ij}=\int_0^1 \bar{m} w_i w_j d\bar{x} \quad (19)$$

$$k_{11ij}=\int_0^1\left[1+\lambda_1\left\{\frac{f}{l^2}-\left(\bar{x}+\bar{b}\right)^2\right\}\right]w_i'' w_j'' d\bar{x}+\lambda_0^2\int_0^1\left\{\frac{f}{l^2}-\left(\bar{x}+\bar{b}\right)^2\right\}w_i' w_j' d\bar{x}+\left\{3g^*\left(1+\frac{h_{12}+h_{32}}{2}\right)^2-\bar{p}\left(\bar{t}\right)\right\}\int_0^1 w_i' w_j' d\bar{x} \quad (20)$$

$$k_{12jl}=-\left(\frac{3}{2}\right)g^* l h_{10} h_{12}\left(1+\alpha\right)\left(1+\frac{h_{12}+h_{32}}{2}\right)\left(\int_0^1 u_l w_i' d\bar{x}\right) \quad (21)$$

$$k_{22kl}=3\times\left(l h_{10}\right)^2\frac{\left(1+\alpha^2 E_{31} h_{31}\right)}{\left(1+E_{31} h_{31}^3\right)}\left(\int_0^1 u_k' u_l' d\bar{x}\right)+\frac{3}{4}g^*\left(l h_{10}\right)^2 h_{12}^2\left(1+\alpha\right)^2\left(\int_0^1 u_k u_l d\bar{x}\right) \quad (22)$$

$$[k_{21}]=[k_{12}]^T \quad (23)$$

In the above, $u_k=\frac{2h_2}{C}\gamma_k$, $u_l=\frac{2h_2}{C}\gamma_l$ and $w_i'=\frac{\partial w_i}{\partial x}$.

The equations (15) and (16) are further simplified to

$$[m]\{\ddot{Q}_1\}+\left[[k]-\bar{P}_0[H]\right]\{Q_1\}-\bar{P}_1\cos\left(\bar{\varpi}\bar{t}\right)[H]\{Q_1\}=\{0\} \quad (24)$$

Where

$$[k]=\left[\bar{k}\right]-[k_{12}][k_{22}]^{-1}[k_{12}]^T \quad (25)$$

$$H_{ij}=\int_0^1 w_i' w_j' d\bar{x} \quad (26)$$

2.3 Free Vibration Analysis

For determination of the natural frequencies $P(t)$ is occupied as zero (i.e., $P_0=P_1=0$) in equation (24). The natural frequencies are obtained from the Eigen values of the $m^{-1}k$ matrix.

3. Result and Discussion

Numerical data are available for parameters such as shear parameter, rotational speed, tapper parameter along width and thickness, core-loss factor. Figures 3 to 8 depict the influence of various system parameters on the structure's free vibration.

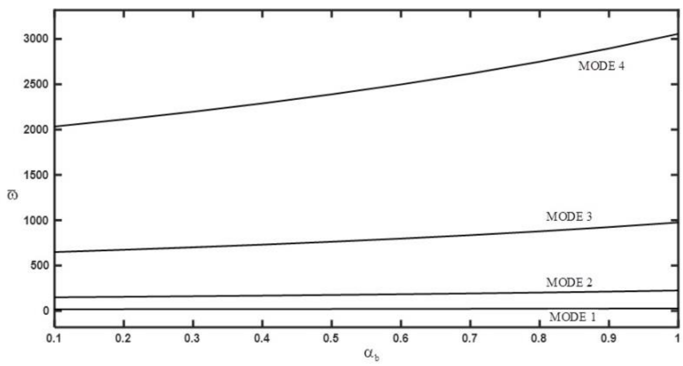

Figure 3 Variation of $\bar{\omega}$ with α_b

Figure 3 depicts the impact of the width taper on the natural frequencies of the system. For larger values of α_b, the natural frequencies of the system increases.

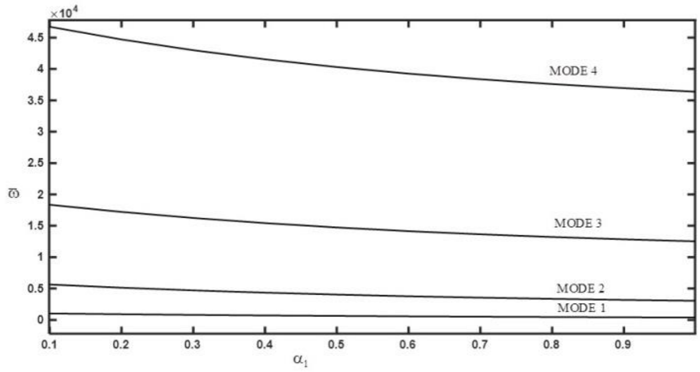

Figure 4 Variation of $\bar{\omega}$ with α_1

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Figure 4 depicts the impact of the taper parameter acting on the thickness on the natural frequencies of the system. For larger values of α_1, the natural frequencies of the system decreases.

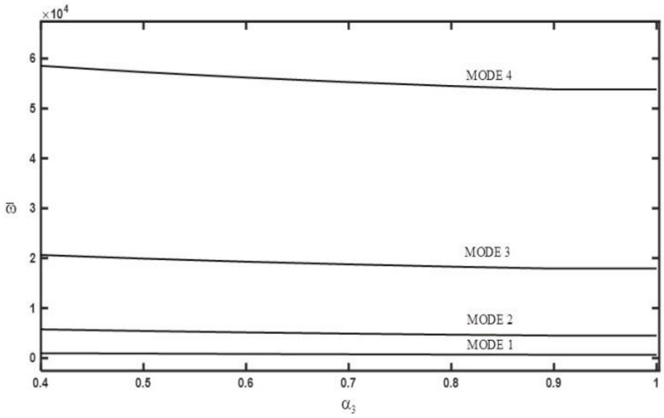

Figure 5 **Variation of $\bar{\varpi}$ with α_3**

Figure 5 depicts the impact of the taper parameter acting on the thickness on the natural frequencies of the system. For larger values of α_3, the natural frequencies of the system decreases

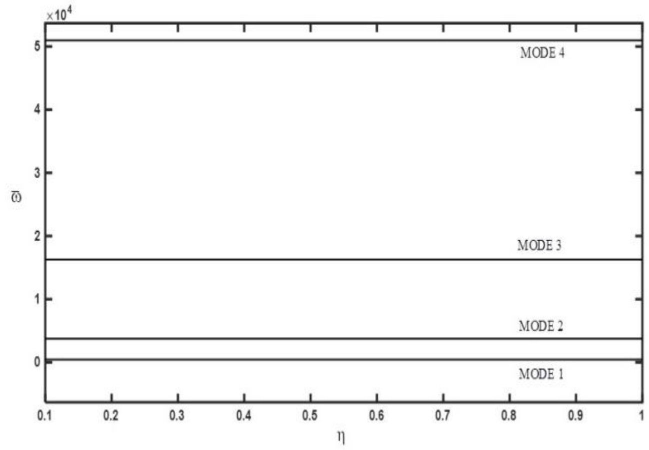

Figure 6 **Variation of $\bar{\varpi}$ with η**

Figure 6 depicts the impact of the core-loss factor on the natural frequencies of the system. For larger values of η, the natural frequencies of the system remains independent with the increase in the value.

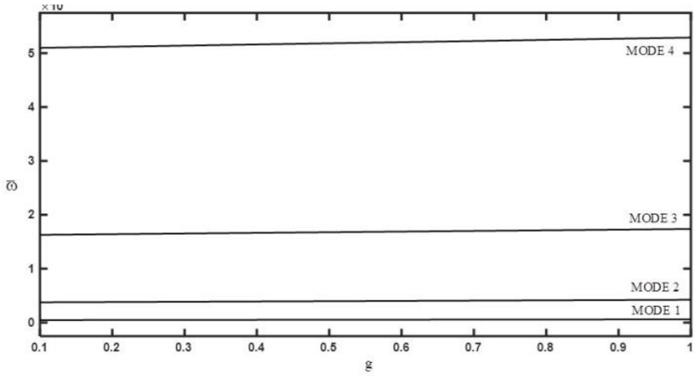

Figure 7 **Variation of $\bar{\varpi}$ with g**

Figure 7 depicts the impact of the shear parameter on the natural frequencies of the system. For larger values of g, the natural frequencies of the system increases. Due to the increment of g the stiffness as well as the shearing behavior of viscos-elastic material increase by which its increase the rigidity of the system and attend stability.

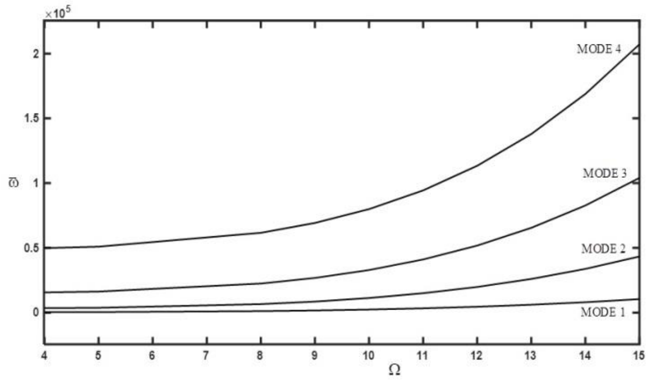

Figure 8 **Variation of $\bar{\varpi}$ with Ω**

Figure 8 depicts the impact of the rotational speed acting on the thickness on the natural frequencies of the system. For larger values of Ω, the natural frequencies of the system increase significantly.

Conclusion

The examination of free vibration analysis of a sandwich beam that's asymmetric, rotating, and tapered across its width and thickness, subjected to pulsating loads, takes into account varying factors such as core-loss factor, taper width, taper characteristics, shear parameter, and rotational velocity. This investigation focuses on the static scenario, considering a clamped-free boundary condition. The deduction is that as the taper width, rotational speed, and shear parameter are heightened, there's a simultaneous increase in the natural frequency across all four modes. As the taper parameter along the thickness is incremented, there's a simultaneous decrease in the natural frequency across all modes and the core loss factor's natural frequency remains independent while increasing.

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