Size Optimization of Truss Structures Based on Genetic Algorithm by Developing Single-point Crossover with Bit-wise Mutation Using FORTRAN Programming

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Abstract
A truss is a collection of axially loaded structural elements connected through pin joints. Optimization of truss structures indicates the determination of best possible conditions that are necessary for achieving the most economical design in terms of lowest possible weights of the truss elements. According to recent studies, Size, Topology and Shape based optimizations are the three possible independent optimization scopes for finding the optimum weight of a truss. The current study focuses on the size optimization of truss structure and aims to identify the most cost-effective sections of the truss using a powerful evolutionary optimization technique named as Genetic Algorithm (GA). Here single point cross over method and bit wise mutation operator are adopted for expanding the search space with the assigned displacement and stress constraints. Fortran based sub routines are developed for each individual steps of GA such as fitness evaluation, selection, cross-over and mutation. Finally the complete program is run for validation of its outcome. While comparing with the solution of an existing literature regarding a minimization of objective function having two design variables the outcomes tend to show a good correlation. Further, the study is extended for the weight minimization of two widely used truss having 10 members and 17 members respectively with standard Finite Element Method of analysis. Minimum weight of each truss is obtained via convergence plot and compared with the existing literatures. With this simple but efficient global optimization technique optimum weight of truss structure can easily be achieved.

Keywords: Bit-wise mutation operator; Finite element method; FORTRAN programming; Genetic algorithm; Single-line crossover

1. Introduction
A truss is a structure that consists of a collection of structural elements connected at pin joints or nodes. It is made up of parts usually constructed from timber or steel that is designed to bear loads by distributing them evenly along the axes of the different parts. Trusses are commonly employed in steel structures such, as, cooling towers, bridges and power transmission towers (Farajpour et al., 2011 ;). Optimization in truss structure design is essential to achieve the minimum possible weight, cost-effectiveness etc. (Lamberti et al. 2003 ;). Here are Various reasons why optimization is crucial in the design of truss structure:

- It provides best designs while managing executing cost and time.
- To provide enough strength to the structure.
- It prescribes minimize weight of structure.

Basic idea of the optimization concept specifically size optimization of truss and the evolutionary optimization technique i.e., Genetic Algorithm (GA) is briefly discussed in the succeeding subsections [1-3].

1.1. Optimization of Truss
Basically, there are three types of truss optimizations to minimize its overall weight namely,
• Size Optimization [Rahami, H. 2008]
• Topology Optimization [Haie & Lee, 1995]
• Shape Optimization [Liu et al. 2016]

In the present study size optimization of truss is considered.

1.1.1 Size optimization
Size optimization purposes determination of the dimensions and proportions of components within a system while considering different performance and design constraints. The main goal of size optimization is to minimize or maximize the cross-sectional area of a structure (Farajpour et.al. 2011 ;) (Figure 1).

![Figure 1 Size Optimization of Truss](image)

1.2. Genetic Algorithm (GA)
Genetic Algorithm (GA) is an evolutionary optimization technique inspired by Darwin's natural selection theory. Developed by Holland in the 1970s, it uses genetic procedures like selection, crossover, and mutation for optimization (Padhey and Simon, 2015). It has vast applications in diverse fields, solving problems in discrete systems (e.g., traveling salesman problem) and continuous systems (e.g., aerospace engineering) (Reynolds, B et.al. 2007 :) or mixed (Lee et al. 2010). The various genetic procedures work parallely to give the best optimized global solution of a given problem (Rajashekkaran and Pai, 2003). It considers the diversity in the search space and finds the global optimum solution. Crossover and Mutation plays an important role in creating a diverse search space (Figure 2).

![Figure 2 Offspring Creation in Cell](image)

2. Method
The basic outlet that describes how GA works and how the 3-processes namely Selection, Crossover and Mutation are implemented (Toğan, V et.al. 2006 :) have been shown in Figure 3.

![Figure 3 Functioning of GA Operators](image)
length, minimum and maximum values, crossover rate, and mutation rate were initialized.

- Binary values of each individual’s chromosome are assigned for the initialization of population

Step 2: Main Loop (Generational Loop)
- Iterate through a predefined number of generations

Step 3: Fitness Evaluation
- Evaluate the fitness of each individual in the population using the objective function.

Step 4: Selection
- Selection of individuals for next generation is performed using Roulette wheel selection process based on their fitness.

Step 5: Crossover
- Single-point crossover with a crossover rate of 0.8 is performed. This involves swapping a portion of the chromosome between two parents to create offspring.

Step 6: Mutation
- Bit-wise mutation with a mutation rate of 0.05 is performed. This involves randomly flipping bits in the chromosome to introduce genetic diversity.

Step 7: Evaluation of New Individuals
- Newly generated individuals are selected for next generation by replacing Less fit individuals.

Step 8: Output
- Print the output for each generation, including the generation number, individuals and fitness.

2.3. Input Parameters
- Total number of design variables = 2
- Total chromosome length = 20
- Population size = 20
- Maximum number of generations = 50
- Crossover probability = 0.8
- Mutation probability = 0.05
- Chromosome length for 1st design variable = 10
- Chromosome length for 2nd design variable = 10
- Minimum and maximum range of 1st design variable = 0 to 6
- Minimum and maximum range of 2nd design variable = 0 to 6

2.2 Size Optimization of Truss
To minimize the weight of truss structure

\[ W(A) = \sum_{i=1}^{n} \rho A_i L_i \]  (3)

Where \( A_i \) is the cross-sectional area of \( i^{th} \) member? \( L_i \) is the length of the \( i^{th} \) member, and \( \rho \) is the weight density of the material and \( n \) is total number of members.

Subject to

\[ g_j(A) \leq 0 \quad j = 1, \ldots, m \]  (4)

Where \( m \) is the number of constraints. The constraints equations are generally expressed in terms of displacement and stress as:

\[ \sigma_j \leq \sigma_u \quad j = 1,2,\ldots,m \]  (5)

\[ v_{kx} \leq v_{ax} \]

\[ v_{ky} \leq v_{ay} \]  (6)

Where \( \sigma_j \) is the stress in \( j^{th} \) member and \( \sigma_u \) is the allowable stress. \( v_{kx} \) and \( v_{ky} \) are vertical displacements at \( k^{th} \) nodes and \( x \) and \( y \) indicate \( x \)-direction and \( y \)-direction and \( v_a \) is the allowable displacement in those two directions.

\[ C = \sum_{j=1}^{m} c_j \]  (7)

If \( g_i(X) > 0 \), then \( c_i = g_i(X) \); or \( g_i(X) \leq 0 \), then \( c_i = 0 \). Where \( m \) is the number of constraints.

And updated objective function is:

\[ \varphi(x) = f(x)(1 + 10C) \]  (8)

And for fitness function:

\[ F_i = [\varphi(x)_{\text{max}} + \varphi(x)_{\text{min}}] - \varphi_i(x) \]  (9)

Where \( i \) is the total number of members (Rajeev, S et al., 1992; and Wu and Chao, 1995).

3. Results and Discussion
3.1. Flowchart of the Proposed Algorithm

The Flowchart for Optimization of Truss Using GA is shown in Figure 4.
3.2. Validation Study

Based on the input parameters considered for the validation study as discussed in the methodology section the convergence plot and output responses are obtained. The convergence plot depicts the convergence of the best fitness value with reference to the generation number and shown in Figure 5. The converged solution is shown in Table 1.

**Figure 4 Flowchart for Optimization of Truss Using GA**

![Flowchart](https://example.com/flowchart.png)

**Figure 5 Convergence Plot for Validation**

![Convergence Plot](https://example.com/convergence_plot.png)
Table 1 Validation Table for Proposed Algorithm

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Maximum generation</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>Best fitness</td>
<td>0.70622</td>
<td>0.95524</td>
</tr>
<tr>
<td>Minimum fitness</td>
<td>0.00663</td>
<td>0.00243</td>
</tr>
</tbody>
</table>

3.2 Optimization of Truss

The optimum weight of truss based on size optimization is obtained from two truss related problems.

3.2.1 Size Optimization of 10 Bar Truss

The first truss problem, which consists of 10 bars, is considered from the study conducted by Rajeev, S and Krishnamurthy, C.S. (1992) (Figure 6). The SI unit system has been adopted. FORTRAN (Rajaraman V., 1997) is used for the FEM coding part of the trusses. For geometry and material properties, readers may follow the stated article. The GA related data are as follows:

a) Population size = 30
b) Maximum number of generations = 50
c) Crossover probability = 0.8
d) Mutation probability = 0.05

Figure 6 10 Bar Truss Structure

The convergence study plot has been shown in Figure 7 and the optimum results are tabulated in Table 2.

Table 2 Output Response Table for 10-Bar Truss

<table>
<thead>
<tr>
<th>Area (sq. cm)</th>
<th>Present study</th>
<th>Rajeev (1992)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>216.14</td>
<td>216.14</td>
</tr>
<tr>
<td>A2</td>
<td>12.84</td>
<td>10.45</td>
</tr>
<tr>
<td>A3</td>
<td>147.75</td>
<td>141.94</td>
</tr>
<tr>
<td>A4</td>
<td>103.23</td>
<td>100.01</td>
</tr>
<tr>
<td>A5</td>
<td>11.61</td>
<td>10.45</td>
</tr>
<tr>
<td>A6</td>
<td>11.61</td>
<td>10.45</td>
</tr>
<tr>
<td>A7</td>
<td>100.01</td>
<td>91.62</td>
</tr>
<tr>
<td>A8</td>
<td>141.94</td>
<td>128.39</td>
</tr>
<tr>
<td>A9</td>
<td>128.39</td>
<td>128.39</td>
</tr>
<tr>
<td>A10</td>
<td>21.81</td>
<td>16.9</td>
</tr>
<tr>
<td>Weight (kg)</td>
<td>2677.13</td>
<td>2546.40</td>
</tr>
</tbody>
</table>

3.2.2 Size optimization of 17 bar truss

The second truss problem, which consists of 17 bars, is considered from the study conducted by (Yaren Aydoğdu et al., 2023) as shown in Figure 8; Here SI unit system has been adopted. For geometry and material properties, readers may follow the stated article. The GA related data are as follows:

a) Population size = 30
b) Maximum number of generations = 100
c) Crossover probability = 0.8
d) Mutation probability = 0.05

Figure 8 17 Bar Truss Structure
The convergence study plot has been shown in Figure 9 and the optimum results are tabulated in Table 3.

**Figure 9 Convergence Plot For 10 Bar Truss**

**Table 3 Output Response Table for 17-Bar Truss**

<table>
<thead>
<tr>
<th>Area (sq. cm)</th>
<th>Present study</th>
<th>Yaren et al. (2023)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>102.78</td>
<td>96.78</td>
</tr>
<tr>
<td>A2</td>
<td>4.97</td>
<td>4.97</td>
</tr>
<tr>
<td>A3</td>
<td>74.65</td>
<td>74.65</td>
</tr>
<tr>
<td>A4</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>A5</td>
<td>57.49</td>
<td>57.49</td>
</tr>
<tr>
<td>A6</td>
<td>36.26</td>
<td>35.03</td>
</tr>
<tr>
<td>A7</td>
<td>76.97</td>
<td>68.78</td>
</tr>
<tr>
<td>A8</td>
<td>2.32</td>
<td>2.32</td>
</tr>
<tr>
<td>A9</td>
<td>54.07</td>
<td>54.07</td>
</tr>
<tr>
<td>A10</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>A11</td>
<td>33.94</td>
<td>33.94</td>
</tr>
<tr>
<td>A12</td>
<td>5.36</td>
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<tr>
<td>A13</td>
<td>39.49</td>
<td>33.94</td>
</tr>
<tr>
<td>A14</td>
<td>26.45</td>
<td>23.29</td>
</tr>
<tr>
<td>A15</td>
<td>34.13</td>
<td>34.13</td>
</tr>
<tr>
<td>A16</td>
<td>5.1</td>
<td>5.1</td>
</tr>
<tr>
<td>A17</td>
<td>39.87</td>
<td>39.87</td>
</tr>
<tr>
<td>Weight (kg)</td>
<td>1252.55</td>
<td>1191</td>
</tr>
</tbody>
</table>

**3.3. Discussion**

This paper includes one validation study and two truss related problems. Convergence plots were utilized to track the optimization process and determine the minimum weight of each truss configuration. The comparison of these weights with those reported in existing literature verify the validity of the optimization results obtained using GA. The simple but efficient nature of our proposed global optimization technique highlights its potential for achieving optimal truss designs with minimal weight, contributing to cost-effectiveness in structural engineering applications. Overall, the discussion underscores the successful application of GA for size optimization of truss structures, demonstrating its efficacy in achieving economically viable designs. The study not only validates the optimization outcomes but also highlights the potential of the proposed approach to streamline the design process and enhance structural performance while minimizing material usage.

**Conclusion**

In conclusion, the optimization of truss structures, particularly the size optimization, has been successfully addressed in this study using the powerful Genetic Algorithm (GA) technique. Through the utilization of single point crossover and bitwise mutation operators within the GA framework, the search space was efficiently explored while adhering to assigned displacement and stress constraints. By developing Fortran-based subroutines for each step of the GA process, including fitness evaluation, selection, crossover, and mutation, a comprehensive and robust optimization approach was established. Validation of the optimization outcomes revealed a strong correlation with existing literature, particularly in scenarios involving minimization of objective functions with two design variables. Furthermore, extension of the study to encompass the weight minimization of two commonly used trusses, each with distinct member configurations, demonstrated the versatility and effectiveness of the proposed technique. Convergence plots elucidated the attainment of minimum weights for each truss, which were then compared with existing literature results, further affirming the credibility and applicability of the developed methodology. Overall, this study underscores the efficiency and efficacy of employing GA as a global optimization technique for achieving...
optimal weights in truss structures, offering a straightforward yet powerful solution to engineering design challenges that has been analyzed in result and discussion section.

References


