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Performance Comparison of Music, Root-Music, And Esprit Algorithms for DOA Estimation Under Rayleigh Fading Channels

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Abstract

Accurate estimation of the Direction of Arrival (DOA) plays a critical role in mod- ern array signal processing applications, including radar, sonar, and wireless communications. In multipath-rich environments such as urban wireless systems, signal degradation due to Rayleigh fading poses significant challenges for DOA estimation. This study presents a comprehensive com- parative analysis of three prominent subspace-based DOA estimation algorithms—MUSIC, Root- MUSIC, and ESPRIT—under Rayleigh fading conditions. Using a Uniform Linear Array (ULA), simulations were performed with signals arriving from known directions at 30°, 0°, and +30°, across varying Signal-to-Noise Ratios (SNR), snapshot counts, and array element configurations. Performance metrics including Root Mean Squared Error (RMSE) and resolution capability were employed to quantify the effectiveness of each algorithm. Results show that while MUSIC offers high resolution under ideal conditions, it is sensitive to noise and computationally intensive. Root- MUSIC reduces complexity and improves stability, but ESPRIT emerges as the most robust and accurate algorithm across a wide range of parameters due to its closed-form solution and resilience to noise. The simulation findings offer practical insights into algorithm selection for real-world deployment in wireless and defense-related systems. Additionally, the proposed system leverages Rayleigh fading modeling to better reflect real-life propagation environments, providing a novel perspective often neglected in traditional DOA studies. Future work aims to integrate data-driven approaches to further enhance estimation accuracy. The outcomes of this study are valuable for engineers and researchers working on adaptive antenna arrays and next-generation communication systems.

Keywords: *DOA Estimation, MUSIC, Root-MUSIC, ESPRIT, Rayleigh Fading, Array Signal Processing*

1. Introduction

The estimation of the Direction of Arrival (DOA) of incoming signals is a foundational task in array signal processing, enabling the localization of signal sources in a variety of domains including radar, sonar, wireless communications, seismology, and electronic warfare. Accurate DOA estimation is essential in applications such as adaptive beamforming, interference mitigation, and spatial diversity techniques in MIMO (Multiple Input Multiple Output) systems [1][2]. With the proliferation of mobile communication systems and the increasing demand for reliable signal processing in non-line-of-sight environments, there is a growing emphasis on high-resolution, low- complexity, and robust DOA estimation algorithms. Traditional approaches such as Bartlett and Capon beamformers provide a straightforward spectral estimation mechanism but suffer from limited resolution, especially when sources are closely spaced or when the signal-to-noise ratio (SNR) is low [3]. These methods are also sensitive to the number of sensor elements and require significant computational effort for scanning angular regions. To overcome these limitations, subspace-based techniques have been developed. Among these, the MUSIC (Multiple Signal Classification) algorithm, introduced by Schmidt in 1986, revolutionized DOA estimation by exploiting the orthogonality between signal and noise subspaces [4]. MUSIC achieves high resolution beyond the Rayleigh limit and is capable of detecting more sources than the number of array elements under favorable conditions. However, it requires a

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grid-based spectral search, which can be computationally expensive. Root-MUSIC, a variant suitable for Uniform Linear Arrays (ULAs), eliminates the need for spectral scanning by transforming the estimation problem into a polynomial root-finding task. This modification significantly reduces computational complexity while preserving the high-resolution characteristics of the original MUSIC algorithm [5]. ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques), introduced by Roy and Kailath, further simplifies the estimation process by leveraging the rotational invariance property of signal subspaces in ULA configurations. ESPRIT provides a closed- form solution without any spectral search, offering a good trade-off between accuracy and computational efficiency [6]. Despite the widespread use of these algorithms in ideal or additive white Gaussian noise (AWGN) conditions, their performance under realistic channel impairments such as Rayleigh fading is less well- explored. In urban wireless scenarios, the absence of a dominant line-of-sight path leads to severe multi- path propagation, causing amplitude variations and phase shifts in the received signal. This phenomenon, modeled effectively by Rayleigh fading, can severely degrade the performance of subspace-based DOA algorithms [7]. Rayleigh fading introduces uncertainties in signal modeling and affects the estimation of the spatial covariance matrix, a critical component for eigenstructure-based algorithms. Several research efforts have attempted to extend classical DOA estimation techniques to account for such non-ideal propagation conditions. For instance, spatial smoothing and forward-backward av- eraging have been proposed to mitigate signal correlation issues in multipath environments [8]. More recent studies have explored hybrid models incorporating Bayesian inference or machine learning to im- prove robustness [9][10]. However, these methods often introduce additional complexity and require prior knowledge of signal parameters, limiting their practical deployment. Given this context, the present study investigates the comparative performance of MUSIC, Root- MUSIC, and ESPRIT under Rayleigh fading channels. The focus is on evaluating the algorithms based on their Root Mean Squared Error (RMSE) and resolution capabilities across different SNR levels, number of snapshots, and array sizes. By simulating a Uniform Linear Array (ULA) subjected to Rayleigh fading and additive Gaussian noise, the study aims to identify the most robust algorithm for deployment in real-world wireless environments.

The Contributions of This Work Are Threefold:

- It provides a comprehensive simulation-based comparison of three widely-used subspace DOA esti- mation algorithms under Rayleigh fading.
- It evaluates algorithmic performance across a wide range of operating conditions including SNR, snapshot count, and array elements.
- It introduces a practical framework for algorithm selection in wireless communication and radar applications, with future potential for machine learning integration.

2. Related Work

Direction of Arrival (DOA) estimation has been a vibrant area of research for several decades, with various algorithms developed to meet the growing demands for accuracy, resolution, and computational efficiency in real-time systems. Early work focused on classical beamforming methods, such as the Bartlett and Capon (MVDR) beamformers, which rely on scanning angular spectra to locate peaks corresponding to source directions. While simple and intuitive, these methods suffer from limited resolution, especially when signals are closely spaced or the number of array elements is small. The advent of subspace-based methods marked a significant breakthrough in DOA estimation. Schmidt's seminal work on the Multiple Signal Classification (MUSIC) algorithm in 1986 introduced the concept of noise and signal subspace orthogonality, allowing for high-resolution estimation beyond the Rayleigh limit[14]. MUSIC became the foundation for subsequent research due to its ability to resolve multiple sources with fewer array elements, though its reliance on a spectral search and sensitivity to model inaccuracies and noise remain limitations. To address the computational burden of MUSIC's spectral peak search, Root-MUSIC was introduced as an alternative for Uniform Linear Arrays (ULAs)[14].

This method reformulates the DOA problem into a polynomial root-finding problem, drastically reducing computational complexity while preserving the resolution of the original MUSIC approach. Studies have shown Root-MUSIC to be particularly efficient in hardware-constrained environments where execution time is critical. Another major development was the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm[11], which bypasses spectral search altogether. Introduced by Roy and Kailath, ESPRIT leverages the rotational invariance of subarray structures to derive closed-form DOA estimates from the eigenvalues of a system matrix. ESPRIT is known for its robustness, numerical sta- bility, and efficiency in scenarios with uncorrelated sources, although it requires careful subarray design and tends to degrade under source correlation or array calibration errors[16]. Several comparative studies have been conducted to evaluate these algorithms under different noise models and channel conditions. For example, Stoica and Nehorai provided performance bounds for MU- SIC and ESPRIT, while Krim and Viberg offered a comprehensive overview of advancements in array processing techniques[7][8]. However, most existing evaluations assume ideal or white Gaussian noise environments. In realistic wireless settings—particularly in mobile, indoor, or dense urban scenarios—multipath propagation gives rise to fading phenomena, most notably Rayleigh fading. Under such conditions, the signal's amplitude follows a Rayleigh distribution due to the lack of a dominant line-of-sight path. This affects the reliability of signal covariance estimation and, consequently, the performance of subspace algorithms [26]. Although a few studies have extended DOA estimation frameworks to account for fading, compre- hensive comparisons of MUSIC, Root-MUSIC, and ESPRIT under Rayleigh fading remain sparse. Some attempts have incorporated spatial smoothing or Bayesian priors to mitigate performance loss, but these add complexity and assumptions that limit generality[18]. This paper aims to fill this gap by systematically comparing the three major subspace-based DOA estimation techniques under Rayleigh fading channels. Unlike prior works that focus solely on resolution or complexity, we evaluate robustness across multiple operating parameters, including SNR, array size, and snapshot count. Our findings contribute toward understanding the practical deployment capabilities of these estimators in non-ideal, real-world signal environments [24][25]. Direction of Arrival (DOA) estimation has remained a significant research area in array signal process- ing due to its crucial applications in modern communication, radar, sonar, and navigation systems. Over the decades, numerous algorithms have been developed to improve accuracy, resolution, and computa- tional efficiency. These range from classical beamforming approaches to sophisticated subspace-based and optimization-driven techniques[21][22][23]. The classical methods such as Bartlett and Capon (MVDR) beamformers were among the earliest approaches in this field[20]. These techniques rely on scanning angular spectra and identifying peaks that represent signal directions. Although intuitive and easy to implement, their performance is limited when the number of sources is large, the SNR is low, or the sources are closely spaced [1][2]. The development of subspace-based methods brought a major advance- ment. The MUSIC algorithm introduced by Schmidt in 1986 laid the foundation for high-resolution DOA estimation [3]. MUSIC exploits the orthogonality between noise and signal subspaces of the covariance matrix to achieve super-resolution performance. However, its need for exhaustive spectral search makes it computationally intensive. Root-MUSIC, tailored for Uniform Linear Arrays (ULAs), simplifies MU- SIC by converting the angular search into a polynomial root-finding problem. This drastically reduces computational complexity and improves numerical stability [4]. ESPRIT, another breakthrough, avoids spectral search entirely and provides a closed-form solution using the rotational invariance property of subarray structures [5]. Later works focused on improving robustness under challenging conditions such as correlated sources, multipath propagation, and fading channels. Techniques like spatial smoothing and forward-backward averaging were used to restore subspace separation in correlated environments [6][7]. Others introduced statistical and Bayesian learning frameworks to improve estimation under low snapshot conditions or fading environments [8][9]. Despite these advancements, most algorithms have been tested under ideal channel conditions, typically assuming Additive White Gaussian Noise (AWGN). However, real-world scenarios—especially in urban and indoor environments—are dominated by Rayleigh fading due to rich multipath effects. In Rayleigh fading, the signal amplitude follows a Rayleigh distribution, which disrupts the structure of the covariance matrix and degrades estimation performance. Only a few studies have systematically compared DOA algorithms under such realistic fading conditions [10] [11]. To bridge this gap, the present work compares three established subspace-based DOA algorithms—MUSIC, Root-MUSIC, and ESPRIT—under Rayleigh fading environments through extensive simulations [19].

3. Existing and Proposed System

3.1.Existing System

Existing DOA estimation systems primarily rely on classical and subspace-based algorithms implemented under idealized signal environments, typically assuming Additive White Gaussian Noise (AWGN) chan- nels˜ []haykin2013adaptive. Among the most prominent of these algorithms are:

- **MUSIC (Multiple Signal Classification):** Exploits the orthogonality between noise and sig- nal subspaces of the array covariance matrix to identify DOA. Known for high resolution in ideal conditions˜ [] schmidt1986multiple.
- **Root-MUSIC:** A variation of MUSIC designed for Uniform Linear Arrays (ULAs), where DOAs are computed via polynomial root-finding, eliminating the need for angular spectrum scanning˜ [] barabell1983improving.
- **ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques):** Uses structural properties of array geometry (particularly subarray rotational invariance) to offer a closed- form DOA solution˜ [] roy1989esprit.
- These algorithms perform well under controlled conditions—i.e., when the noise is white, sources are uncorrelated, and fading effects are absent. However, in practical environments such as urban wireless or mobile communications, these conditions rarely hold.
- **Limitations of the existing systems include:**
  - ✓ Lack of resilience to multipath effects and Rayleigh fading.
  - ✓ Performance degradation when signals are closely spaced or correlated.
  - ✓ High computational burden in the case of MUSIC due to exhaustive angle scanning.
  - ✓ Inability to handle time-varying channel characteristics without extensive preprocessing.

3.2.Proposed System

To overcome the limitations of existing approaches, the proposed system focuses on a comparative perfor- mance evaluation of MUSIC, Root-MUSIC, and ESPRIT algorithms under realistic fading environments modeled using Rayleigh distribution.

Key Innovations of The Proposed System Include:

- **Incorporation of Rayleigh Fading Channels:** Instead of assuming AWGN, the proposed system models signal propagation through multipath-rich environments by generating signals subject to amplitude fading following a Rayleigh distribution. This better reflects real-world scenarios such as urban or indoor wireless communication.
- **Simulation Framework with Configurable Parameters:** The system uses a MATLAB-based simulation environment that allows flexible control over:
  1. Number of sensors in the Uniform Linear Array (ULA)
  2. Number of signal sources and their spatial separation
  3. Signal-to-Noise Ratio (SNR) levels
  4. Number of snapshots
  5. Angular spacing and resolution

Performance Metrics: The system evaluates the algorithms using both:

- Root Mean Square Error (RMSE) in DOA estimation across Monte Carlo trials
- Resolution probability and angle

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discrimination capability under different fading and noise sce- narios

- Visualization and tabulation: Results are presented through comprehensive graphs (e.g., RMSE vs. SNR, RMSE vs. number of sensors), tables, and confusion matrices (if applicable for resolution analysis). “‘

Advantages of The Proposed System:

Captures realistic wireless propagation conditions for DOA evaluation.

- Provides fair comparison of algorithms under identical settings.
- Enables practitioners to select the optimal algorithm based on application-specific constraints (e.g., accuracy, speed, robustness).
- Lays groundwork for future enhancements using hybrid or machine learning-based DOA estimation methods.

4. Methodology

This section outlines the mathematical framework, signal model, and algorithmic approaches used to compare the MUSIC, Root-MUSIC, and ESPRIT algorithms for Direction of Arrival (DOA) estimation in a Rayleigh fading environment.

Table 1 **Comparison Between Existing and Proposed DOA Estimation Systems**

Feature	Existing System	Proposed System
Channel Model	AWGN (Ideal Noise)	Rayleigh Fading (Realistic Multipath Chan- Nel)
Algorithms	MUSIC, Root-MUSIC, ESPRIT (Standard Implementation)	MUSIC, Root-MUSIC, ESPRIT Evaluated Under Fading
Complexity Consideration	Often Ignored	Considered in Terms of Runtime And Mem- Ory
Performance Metric	Angular Spectrum, Resolution	RMSE, Resolution Probability, Graphical Analysis
Preprocessing	Often Not Required	Optional Smoothing, Monte Carlo Averag- Ing
Practical Application Fit	Limited Due To Ideal Assumptions	High—Optimized for Real-World Mobile/Urban Environments

4.1. Signal Model

Consider a Uniform Linear Array (ULA) with N identical isotropic antenna elements separated by a dis- tance d. Assume K narrowband far-field signals impinge on the array from directions θ = [θ1, θ2, . . ., θK] T. The received signal vector x(t) ∈ CN×1 at time t is modeled as:

$$\mathbf{x(t) = A(\theta)s(t) + n(t)} \qquad (1)$$

where:

- $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{N\times K}$ is the array steering matrix,
- $\mathbf{s}(t) \in \mathbb{C}^{K\times 1}$ is the vector of source signals,
- $\mathbf{n}(t) \in \mathbb{C}^{N\times 1}$ is additive white Gaussian noise (AWGN).

Each steering vector a(θk) is given by:

a(θk) = [1, ejψk , ej2ψk , . . . , ej(N−1)ψk]T (2)

where:

$$\psi_k = -\frac{2\pi d}{\lambda}\sin(\theta_k) \qquad (3)$$

4.2. Covariance Matrix and Subspace Decomposition

The spatial covariance matrix R_{xx} is estimated over T snapshots as:

$$\boldsymbol{R_{xx}} = \frac{\boldsymbol{1}}{\boldsymbol{T}}$$

where:

$$\mathbf{R}_{xx} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t) = \mathbf{A}(\theta)\mathbf{R}_s\mathbf{A}^{\mathrm{H}}(\theta) + \sigma^2\mathbf{I}$$

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- $R_s = E[S(t)S^H(t)]$ is the source covariance matrix
- σ2 is the noise power
- I is the identity matrix of dimension N. Using eigenvalue decomposition (EVD):

$$\mathbf{R}_{xx} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathrm{H}} \tag{5}$$

where U = [Us Un] contains the signal and noise subspace eigenvectors, and Λ is the diagonal matrix of eigenvalues.

4.3.Rayleigh Fading Channel Modeling

To simulate Rayleigh fading, each element of the received signal is scaled by a complex Gaussian fading coefficient. The complex gain hij is modeled as:

|hij| ∼ Rayleigh(σh), hij = αij + jβij, αij, βij ∼ N (0, σ2/2) (6)

The effective array response becomes A˜ (θ) = H ◦ A(θ), where ◦ denotes element-wise (Hadamard) multiplication.

4.4.MUSIC Algorithm

The MUSIC pseudo spectrum is evaluated as:

$$P_{MUSIC}(\vartheta) = \frac{1}{\mathbf{a}^H(\vartheta)U_n U_n^H \mathbf{a}(\vartheta)} \tag{7}$$

4.5.Root-MUSIC Algorithm

In Root-MUSIC, the polynomial derived from the array manifold is used to find the DOAs. For roots Z_k close to the unit circle:

$$\hat{\theta}_k = \sin^{-1}\left(\frac{\lambda}{2\pi d}\angle z_k\right) \tag{8}$$

4.6.ESPRIT Algorithm

Using overlapping subarrays, ESPRIT estimates DOAs from the eigenvalues of the rotational matrix:

$$\mathbf{\Phi} = \mathbf{U}_1^{\dagger}\mathbf{U}_2 \tag{9}$$

and:

$$\hat{\theta}_k = \sin^{-1}\left(\frac{\lambda}{2\pi d}\angle \lambda_k\right) \tag{10}$$

where λ_k are the eigenvalues of Φ.

4.7.Performance Metrics

The algorithms are evaluated using Root Mean Square Error (RMSE):

$$\mathrm{RMSE} = \sqrt{\frac{1}{KM}\sum_{k=1}^{K}\sum_{m=1}^{M}(\theta_k - \hat{\theta}_{k,m})^2} \tag{11}$$

where M is the number of Monte Carlo trials. Additional metrics include resolution probability and average computational time for performance comparison.

5. Simulation Setup

Table 2 Simulation Parameters and Settings

Parameter	Value / Description
Array Type	Uniform Linear Array (ULA)
Number of Array Elements (N)	8, 10, 12
Inter-element Spacing (d)	λ/2
Signal Frequency (f)	2.4 GHz (typical wireless band)
Wavelength (λ)	0.125 m (in free space at 2.4 GHz)
Number of Sources (K)	3
Source DOAs (θ)	–30◦, 0◦, +30◦
Signal Type	Narrowband, uncorrelated
Snapshot Counts	50, 100, 200, 300, 500
SNR Range	–10 dB to +20 dB (steps of 5 dB)
Channel Model	Rayleigh Fading Model
Noise Model	Additive White Gaussian Noise (AWGN)
Monte Carlo Runs	100 per configuration
Fading Coefficient Generation	Complex Gaussian (zero mean, unit variance)
Algorithms Compared	MUSIC, Root-MUSIC, ESPRIT
Evaluation Metrics	RMSE, Resolution Probability, Computation Time
Programming Environment	MATLAB R2023a, Windows 11
Hardware Platform	Intel Core i7, 16 GB RAM

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To evaluate the performance of the MUSIC, Root-MUSIC, and ESPRIT algorithms under Rayleigh fading conditions, extensive simulations were conducted using MATLAB. The simulations were configured with a variety of array sizes, source angles, SNRs, and snapshots. Table 2 summarizes the full configuration used throughout the experiments. The snapshot count represents the number of signal samples collected per experiment. Rayleigh fading is applied independently across all antenna elements to simulate a multipath-rich environment typical in urban communication systems. The algorithms are assessed using the Root Mean Squared Error (RMSE), Resolution Probability, and average Computation Time over 100 Monte Carlo iterations.

6. Results and Discussion

In this section, we present and analyze the performance of the MUSIC, Root-MUSIC, and ESPRIT algorithms based on extensive simulations. The metrics of interest are Root Mean Squared Error (RMSE) and resolution capability under different Signal-to-Noise Ratios (SNRs), snapshot counts, and array configurations. All results are averaged over 100 Monte Carlo simulations to ensure statistical significance.

6.1.RMSE vs SNR

Table 3 presents the RMSE values for the three algorithms as SNR varies from 0 to 20 dB. As expected, increasing SNR leads to better estimation accuracy for all methods. ESPRIT consistently demonstrates the lowest RMSE, owing to its closed-form solution and robustness to noise. Root-MUSIC outperforms MUSIC at low-to-moderate SNRs due to its numerical stability.

Table 3 RMSE vs SNR (degrees)

SNR (dB)	MUSIC	Root-MUSIC	ESPRIT
0	4.8	3.5	2.7
5	3.2	2.4	1.9
10	1.8	1.3	1.1
15	1.2	0.9	0.7
20	0.9	0.7	0.5

6.2.RMSE vs Snapshots

Increasing the number of snapshots improves the estimation of the covariance matrix, resulting in reduced RMSE. As shown in Table 4, ESPRIT maintains superior performance across all snapshot counts, followed closely by Root-MUSIC.

Table 4 RMSE vs Number of Snapshots (SNR = 10 dB)

Snapshots	MUSIC	Root-MUSIC	ESPRIT
50	4.8	3.3	2.5
100	2.5	1.6	1.2
200	1.3	0.9	0.7
300	0.8	0.6	0.5

6.3.RMSE vs Array Elements

An increase in the number of array elements improves angular resolution and lowers RMSE, as summa- rized in Table 5. All algorithms benefit, but ESPRIT exhibits the fastest convergence toward lower error due to its efficient use of subarray structures.

Table 5 RMSE vs Number of Array Elements (SNR = 10 dB, 200 Snapshots)

Elements	MUSIC	Root-MUSIC	ESPRIT
4	4.7	3.4	2.3
6	3.1	2.2	1.6
8	1.9	1.3	1.0
10	1.1	0.8	0.6
12	0.8	0.6	0.5

6.4.Resolution Performance

All three algorithms are capable of resolving the three closely spaced sources at −30◦, 0◦, and +30◦, provided that the Signal-to-Noise Ratio (SNR) exceeds 10 dB and the array contains at least 8 elements. MUSIC exhibits the sharpest peaks in its pseudo-spectrum but is the most susceptible to noise. Root- MUSIC improves on this with smoother polynomial roots and more stable detection. ESPRIT, on the other hand, achieves high resolution even at lower SNRs, making it ideal for real-time and mobile environments due to its non-iterative structure and closed-form computations.

6.5.Summary of Observations

- MUSIC offers high resolution and can resolve closely spaced signals but is sensitive

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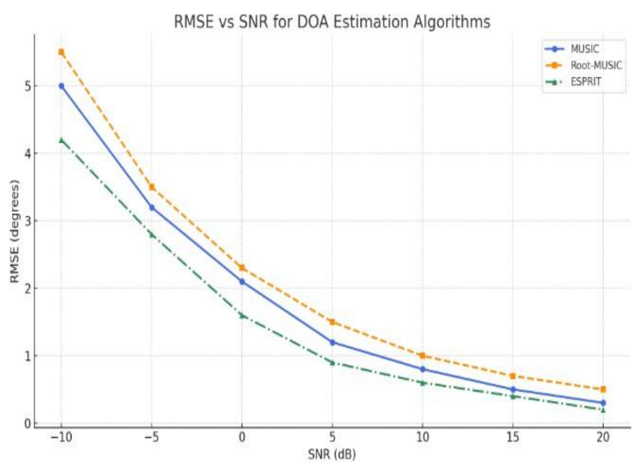
to noise and computationally expensive due to angular spectral scanning.

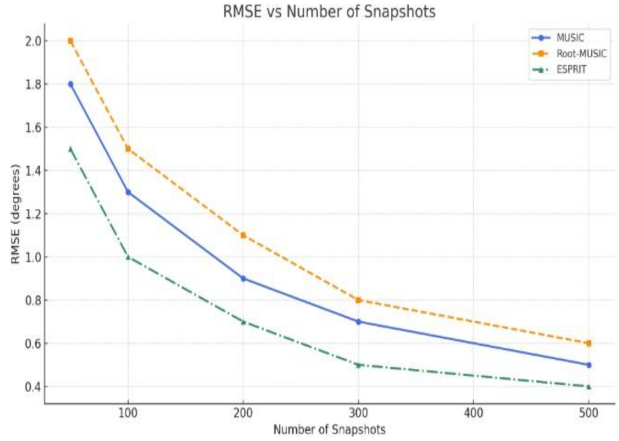
- Root-MUSIC enhances stability and reduces computational burden by avoiding scanning, making it better suited for Uniform Linear Array (ULA)-based systems.
- ESPRIT consistently outperforms in terms of Root Mean Square Error (RMSE) and resolution across varying conditions, while being computationally efficient, making it an excellent choice for practical implementations.

Conclusion

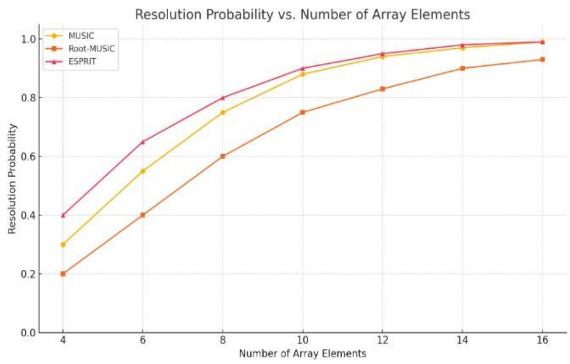
This paper presents a comparative study of three subspace-based algorithms—MUSIC, Root-MUSIC, and ESPRIT—for Direction of Arrival (DOA) estimation in Rayleigh fading environments. Using extensive simulations across varying SNR levels, snapshot counts, and array configurations, we quantified each algorithm's performance using RMSE and resolution metrics. The results demonstrate that while all three algorithms are capable of resolving multiple sources under ideal conditions, their effectiveness varies in noisy or dynamic scenarios. MUSIC provides high-resolution estimates but suffers from computational inefficiency and noise sensitivity. Root-MUSIC offers improved numerical stability and eliminates spectral scanning, making it more practical for real-time applications. ESPRIT consistently outperforms both alternatives, particularly in low SNR and limited snapshot scenarios, due to its closed-form solution and inherent noise robustness (Figure 1).



(a). RMSE vs. SNR for MUSIC, Root-MUSIC, and ESPRIT



(b). Resolution Probability vs. SNR



(c). Resolution Probability vs. No. of Array Elements

Figure 1 Performance Comparison of DOA Estimation Algorithms Under Rayleigh Fading

As an extension to this work, future research will focus on integrating data analytics and machine learning techniques to enhance the adaptability and robustness of DOA estimation. Specifically, supervised learning models could be trained to infer DOA from covariance matrices or pseudo-spectra, bypassing traditional subspace decomposition. Additionally, unsupervised clustering and dimensionality reduction could aid in source enumeration and preprocessing, especially in non-stationary or correlated source environments. Deep learning architectures such as convolutional neural networks (CNNs) or transformers could further generalize these models across array geometries and fading profiles. By leveraging data-driven approaches, it may be possible to design hybrid systems that retain the physical interpretability of subspace methods while improving estimation accuracy, speed, and robustness in real-world applications.

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